Uva Wellassa University, Sri Lanka
Btech. Science and Technology
End Semester Examination- Semester 1
January -2009

SCT 201-2 – Mathematics 1

Answer for Four (04) questions only **Time: Two hours**
No. of questions: Six (06)
Calculators are acceptable
All expressions are given in standard mathematical notations
You may use standard notations
Any new notation or abbreviations you use must be clearly defined

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1. a. Write the negations of following mathematical statements.

   i. $x = y$

   ii. $x < 5$

   (05 marks)

   b. Given that, A:- Set of real numbers
   B:- Set of complex numbers

   Then determine the truth value of following statements.

   i. $\mathbf{A} \subset \mathbf{B}$
   ii. $\mathbf{A} = \mathbf{B}$
   iii. $\mathbf{A} \in \mathbf{B}$
   iv. $x, y \in \mathbf{A} \Rightarrow x + iy \in \mathbf{B}$

   (08 marks)

   c. Construct truth tables and prove the following logical equivalences.

   i. $p \vee q \equiv \sim(\sim p \wedge \sim q)$

   ii. $(p \Rightarrow q) \vee (q \Rightarrow p) \equiv \sim p \vee q \vee \sim q \vee p \equiv T$

   (12 marks)

2. a. Determine the following limits.

i. $$\lim_{x \to 3} \frac{x^2 + 1}{x + 2}$$

(02 marks)

ii. $$\lim_{x \to 1} \frac{2 \ln x}{x - 1}$$

(04 marks)

iii. $$\lim_{x \to 0} \frac{e^x - 1}{x}$$

(04 marks)

b. At what points, the function $f(x)$ is discontinuous, if

$$f(x) = \frac{1}{x - 3} + 3x$$

(05 marks)

c. Determine whether the function $g(x)$ is continuous at $x = 1$.

$$g(x) = \begin{cases} 3x - 5 \ , & if\ x \neq 1 \\ 2 & , \ if\ x = 1 \end{cases}$$

(10 marks)

3. a. In a study it was found that the temperature of sea water T at time t (measured in days) at a depth of x (measured in feet) can be modeled by the following function,

$$T(x, t) = T_0 + T_1 e^{-\lambda x} sin(\omega t - \lambda x).$$

where; $\omega = \frac{2\pi}{365}$ and λ, T_0 and T_1 are positive constants.

i. Find the rate of change of temperature with respect to the depth.

(05 marks)

ii. Find the rate of change of temperature with respect to the time.

(05 marks)

iii. Show that $T(x, t)$ satisfies the heat equation $T_t = k\ T_{xx}$ for a certain constant k.

(10 marks)

b. The gas law for a fixed molar amount n of an ideal gas at absolute temperature T, pressure P and volume V is **V = nRT**, where R is the gas constant. Show that;

$$\frac{\partial P}{\partial V} \frac{\partial V}{\partial T} \frac{\partial T}{\partial P} = -1$$

(05 marks)

4. a. i. Express $\frac{1}{(1+x)}$ as a power series.

(05 marks)

ii. Differentiate the answer of (i) with respect to x and hence obtain a power series for $\frac{1}{(1+x)^2}$.

(05 marks)

b. The weight $\omega(x)$ (force due to gravity) of an object of mass m varies with it's altitude (length) x miles above the surface of the earth according to the function,

$$\omega(x) = \frac{mgR^2}{(R+x)^2},$$

Where R is the radius of the earth and g is the acceleration due to gravity.

i. Express $\omega(x)$ as a infinite series.

(04 marks)

ii. When x is much smaller than R, show that $\omega(x) \cong mg(1 - \frac{2x}{R})$.

(06 marks)

c. i. Find the Maclaurin series for $f(x) = e^x$.

(05 marks)

5. a. Prove that $|\sin x| \leq |x|$ for all real x.

(05 marks)

b. What is the area bounded by the curves $y = x^2 - 4x$ and $y = 2x$.

(08 marks)

c. Find the volume of the solid generated by rotating the curve $y = 2x^3$, between $y = 0$ and $y = 4$ about the $y-$axis.

(12 marks)

6. a. Show that the equation $x^3 + x - 1 = 0$ has at least one solution (root) in the interval (0, 1).

(05 marks)

b. Suppose $f(x) = \sum_{n=0}^{\infty} c_n x^n$ is an odd function. Then show that,

$$c_0 = c_2 = c_4 = \cdots \ldots\ldots\ldots\ldots = 0$$

(05 marks)

c. Calculate the values for,

i. $(1+i)^{10}$

(05 marks)

ii. $(1+i)^{1/5}$

(10 marks)