

**Instructions to candidates**

**Duration:** Two(02) hours

**Number of questions:** Four(04) Essay Questions

**Mark allocation:** 100 marks

Use standard symbols without definition.

Scientific calculators are allowed.

**Answer all questions.**

1.

a. Let  $A = \begin{pmatrix} 2 & 8 \\ -10 & 5 \end{pmatrix}$ ,  $B = \begin{pmatrix} -9 & 4 \\ 3 & -2 \end{pmatrix}$  and  $C = \begin{pmatrix} 2 & -1 & 4 \\ 4 & 0 & -5 \\ 7 & 2 & 6 \end{pmatrix}$

Find;

i.  $2A + B$  (02 mark)

ii.  $A - 3B$  (02 mark)

iii. transpose of matrix  $2C$  (02 mark)

iv. product of  $A$  and  $B$  ( $A \times B$ ) (03 mark)

b. Find the minors, cofactors, adjoint matrix and inverse matrix of the following matrix  $D$ .

$$D = \begin{pmatrix} -1 & 2 & 0 \\ 3 & 5 & 1 \\ 2 & -2 & 3 \end{pmatrix}$$

(10 mark)

c. Use Cramer's Rule to solve the following system of linear equations.

$$x + 3y - 4z = 9$$

$$2x - y + z = 3$$

$$x + 2y - 5z = 16$$

(06 mark)

2.

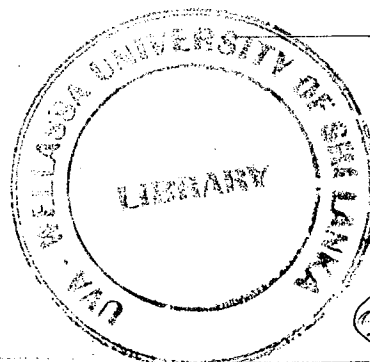
a. Evaluate the following limits.

i.  $\lim_{x \rightarrow 2} \left( \frac{x^2 - 7x + 10}{x - 2} \right)$  (02 mark)

ii.  $\lim_{x \rightarrow 2} \left( \frac{4 - \sqrt{18 - x}}{x - 2} \right)$  (02 mark)

iii.  $\lim_{x \rightarrow \infty} \left( \frac{4x^{48} - 5x^{10} + 6x^5 + 3}{-x^{48} + 3x + 4} \right)$  (02 mark)

iv.  $\lim_{x \rightarrow -3} \left( \frac{x^5 + 243}{x^4 - 81} \right)$  (02 mark)



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b. Differentiate following functions with respect to  $x$ .

i.  $y = x^{10} + \frac{1}{12}x^6 + 5x^4 + \frac{1}{2}x^2 + 6$  (02 mark)

ii.  $y = \ln(5x^3 + x^2 + 6x + 4)$  (02 mark)

iii.  $y = \frac{(3x+5)^5}{(x^3+2x^2-3x+5)^2}$  (03 mark)

iv.  $y = e^{(x^2+2x+3)} \ln 2x$  (03 mark)

c. Let  $f(x) = 4x^4 - 3x^3 + 6x^2 + 9x + 1$  then find  $f'(2)$  and  $f''(2)$  (04 mark)

d. Sketch the graph of  $f(x) = 2x^3 - 3x^2 - 12x + 5$  by using the first derivative test. (08 mark)

3.

a. Integrate the following functions with respect to  $x$ .

i.  $\int (2x^3 + 3x - 1) dx$  (02 mark)

ii.  $\int (8x + 5)^6 dx$  (02 mark)

iii.  $\int \frac{6x^2+16x-6}{x^3+4x^2-3x+5} dx$  (02 mark)

iv.  $\int \frac{1}{\sqrt{16-(x-1)^2}} dx$  (02 mark)

v.  $\int \sec^2\left(\frac{1}{2}x\right) dx$  (02 mark)

b. Evaluate the following definite integrals.

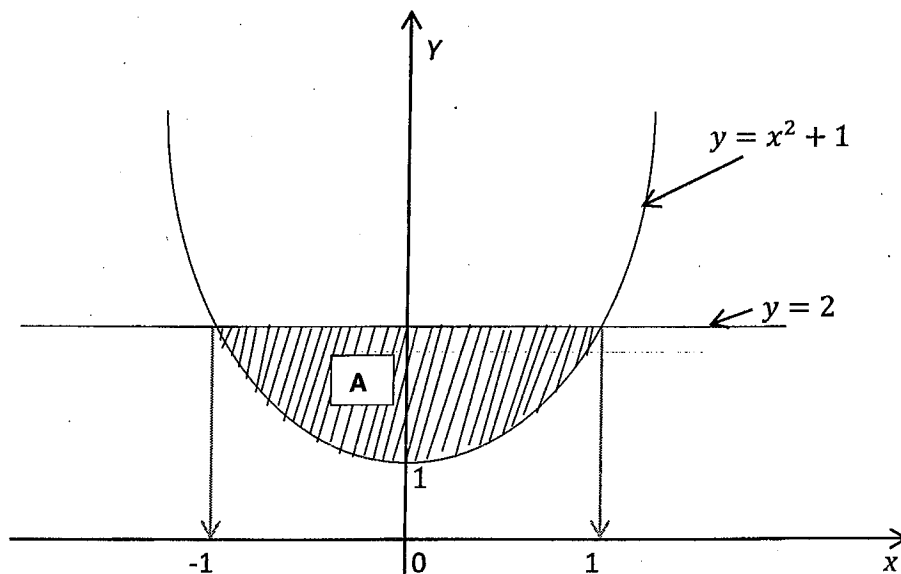
i.  $\int_2^5 (5x + 3)^2 dx$  (03 mark)

ii.  $\int_0^1 x \ln x dx$  (03 mark)

c. Compute the below integral, using integrate by parts.

$I = \int x^2 e^x dx,$  (08 mark)

- d. Find the area of the region  $A$  (colour area) in the figure 01 that is bounded by the curve  $y = x^2 + 1$  and the line  $y = 2$ . (06 mark)



4.

- a. Determine whether the following sequences are convergent or divergent. Justify your answer.

- i.  $\left\{\frac{1}{n^2}\right\}_{n=1}^{\infty}$  (02 mark)
- ii.  $\{3n^2 + 1\}_{n=1}^{\infty}$  (02 mark)
- iii.  $\left\{\frac{2n+1}{n^2+n-3}\right\}_{n=1}^{\infty}$  (03 mark)
- iv.  $\left\{\frac{(-1)^{n+1}}{n^2+1}\right\}_{n=1}^{\infty}$  (02 mark)

- b. Determine whether each of the following series are convergent or divergent. Justify your answer.

- i.  $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$  (03 mark)
- ii.  $1 + 6 + 36 + 216 + 1296 + \dots$  (03 mark)

