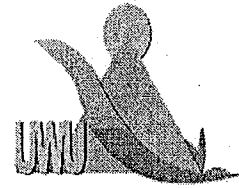


Uva Wellassa University, Sri Lanka.

End Semester Examination – June 2009

AAS 2104-2 Mathematics for biological sciences



Time : Two (2) hours

(Repeat Examination)

Answer all questions

1. Point out the difference between an algebraic expression and an algebraic equation.

(2 marks)

a) A plastic water pipe is used to supply water to a paddy field, the pipe has a hollow center of 100 inches long, 1 inch thick, and has an inner radius of x inch. Represent the volume of the plastic used to construct the pipe and simplify it. (Given that, the volume V of a right circular cylinder of radius r and height h is given by $V = \pi r^2 h$)

(7 marks)

b) A nursery bed has the shape of a rectangle surmounted by a semicircle. If the perimeter of the Nursery bed is 30 feet. Express the area of the Nursery bed as a function of the width x of it.

(7 marks)



c) A cubic container for shipping Orange is formed by coating a metal mold with polystyrene inside. If the metal mold is a cube with sides x cm long, and the polystyrene coating is 2cm thick. Write an algebraic expression in terms of x that represents the volume of the polystyrene used to construct the container. If x is given as 20cm, simplify the expression. (Volume of a cube with sides of length l is l^3)

(7 marks)

2. What is the major difference between an independent variable and a dependent variable. (2 marks)

a) The number of plants N (in thousands) of an invasive species in an island is shown in the table. (Mid year estimates are given.)

t	1991	1993	1995	1997
N	132	304	873	2461

- Use data to sketch a rough graph of N as a function of t .
- Use your graph to estimate the number of plants in the island at midyear in 1994 and 1996. (7 marks)

b) A poultry farmer knows that it costs Rs9,000 to farm 1000 chickens and Rs12,000 to farm 1500 chickens.

- Express the cost as a function of the number chicken farmed, assuming that it is linear. Then sketch the graph.
- What is the slope of the graph and what does it represent.
- What is the y -intercept of the graph and what does it represent. (8 marks)

c) In a certain country, harvest tax is assessed as follows. There is no tax on harvest income up to Rs10,000. Any harvest income over Rs10,000 up to Rs20,000 is taxed at a rate of 10%. Any harvest income over Rs20,000 is taxed at 15%.

- Sketch the graph of the tax rate (R) as a function of the harvest income (I).
- How much tax is assessed on a harvest income of Rs26,000.
- Sketch the graph of the total assessed tax (T) as a function of income (I). (8 marks)

3. What is the difference between inequality and equality. (2 marks)

a) An individual wants to use milk and orange juice to increase the amount of calcium and vitamin-A in her daily diet. An ounce of milk contains 30 mg of calcium and 50 mg of vitamin-A. An ounce of orange juice contains 5 mg of calcium and 60 mg of vitamin-A. How many ounce of milk and orange juice should he drinks each day to provide exactly 325 mg of calcium and 800 mg of vitamin A. (7 marks)

b) Packed tea should be kept between $68^{\circ}F$ and $77^{\circ}F$. Find the range in degrees of Celsius if the Celsius/Fahrenheit convention formula is $F = \frac{9}{5}C + 32$. (7 marks)

c) If bacteria in a certain culture doubles every $\frac{1}{2}$ hours, write an equation that gives the number of bacteria N in the culture after t hours, assuming the culture has 100 bacteria at the start.

- i. Find number of bacteria after 3 hours.
- ii. Graph the equation for $0 \leq t \leq 5$ and show the value obtained by the above part.

(7 marks)

4. Point out the major purpose of having differentiation.

(2 marks)

a) Suppose that the population for a certain bacteria $B(t)$ is given by,

$$B(t) = 2000 + 3t + 2t^2 + 1.5t^3 \quad ; \quad \text{where } t \text{ is the time by seconds}$$

- i. Find and interpret $B(0)$.
- ii. Find $\frac{dB}{dt}$, when $t = 20s$, and interpret it.

(8 marks)

b) In a fish farm, a population of fish is introduced into a pond and harvested regularly. A model for the rate of change of the fish population is given by,

$$\frac{dP}{dt} = r_0 \left(1 - \frac{P(t)}{P_c}\right) P(t) - \beta P(t)$$

Where r_0 is the birth rate of the fish, P_c is the maximum population that the pond can sustain, β is the percentage of the population that is harvested and $P(t)$ is the population at time t .

- i. What is the value of $\frac{dP}{dt}$ corresponds to a stable population.
- ii. If the pond can sustain 10,000 fish, the birth rate is 15%, and the harvesting rate is 4%, find the stable population level.
- iii. What happens if β is raised to 5%.

(9 marks)

c) A large fish tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then the volume V of water remaining of the tank after t minutes as,

$$V(t) = 5000 \left(1 - \frac{t}{40}\right)^2 \quad ; \quad 0 \leq t \leq 40$$

- i. Find the rate at which the water drains from the tank at any time t .
- ii. Find the draining rate at 5th minute.
- iii. At what time the flowing is fastest.
(Hint: - Graph $V'(t)$ with respect to t)
- iv. At what time the flowing is slowest.

(10 marks)

