Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
200 Level 2nd Semester Examination – Dec./Jan. 2017/18
SCT 202-3 Differential Equations & Applications

Uva Wellassa University

Instructions to candidates
Duration: Two (02) hours
Number of questions: Four (04) Essay Questions
Mark allocation: 120 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions

PART A

1.

a. Determine whether $y(x)=2e^{-x}+xe^{-x}$ is a solution of $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$. (04 mark)

b. Find the general solution of $\frac{dy}{dx}=\frac{x^2+1}{y^2+y}$ by *separating the variables.* (04 mark)

c. Show that the following differential equation is homogeneous and solve by substituting $y=vx$, where v is a function of x. (06 mark)

$$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}.$$

d. Find an integrating factor for $x\frac{dy}{dx}+y=x^3$, and hence determine the general solution of the differential equation. (06 mark)

e. Solve the Bernoulli equation $\frac{dy}{dx}+\frac{1}{x}y=xy^2$. (07 mark)

f. Five mice in a stable population of 500 are intentionally infected with a contagious disease to test a theory of epidemic spread that postulates the rate of change in the infected population is proportional to the product of the number of mice who have the disease with the number that are disease free. That is,
Let $N(t)$ denote the number of mice with the disease at time t. It is given that $N(0)=5$, and it follows that $500-N(t)$ is the number of mice without the disease at time t. The theory predicts that

$$\frac{dN}{dt}=KN(500-N).$$

Assuming the theory is correct, how long will it take half the population to contract the disease? (08 mark)

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2.

a. Find the general solution of the following second order differential equations.

i. $\frac{d^2y}{dx^2}+10\frac{dy}{dx}+21y=0$ (04 mark)

ii. $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+16y=0$ (04 mark)

iii. $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=\sin 2x$ (06 mark)

iv. $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+25y=64e^{-x}$ (06 mark)

b. The simple RLC electrical circuit shown in figure 2.1 consists of a resistor $R=10$ ohms; a capacitor $C=10^{-2}$ farads; an inductor $L=0.5$ henries; and electromotive force (emf) $E(t)=12$ volts, usually a battery or a generator, all connected in series. The current I flowing through the circuit is measured in amperes and the charge q on the capacitor is measured in coulombs.

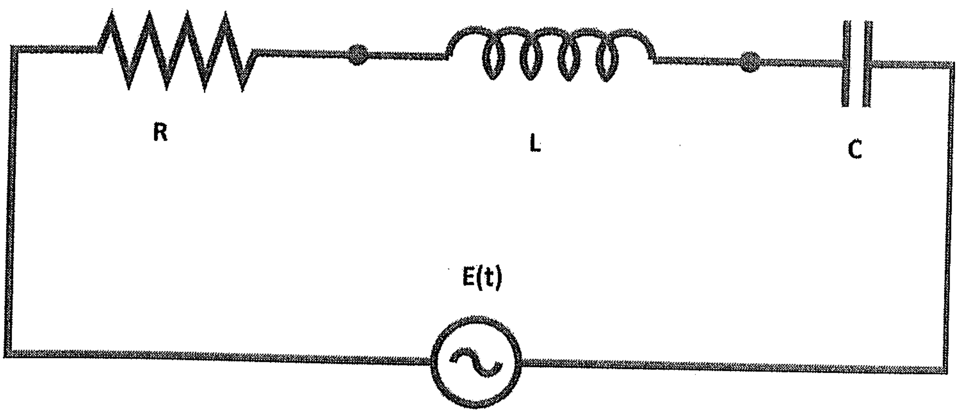

Figure 2.1

From the *Kirchhoff's loop law*, the relationship between I and t is

$$\frac{d^2I}{dt^2}+\frac{R}{L}\frac{dI}{dt}+\frac{1}{LC}I=\frac{1}{L}\frac{d\,E(t)}{dt}.$$

Assuming no initial current at $t=0$ when the voltage is first applied, find the subsequent current $I(t)$ in the system. (08 mark)

c. Using *D-operator method*, find the general solution of $(D^2-3D+2)y=3\sin 2x$. (07 mark)

3.

a. Determine the general solution of the following system of linear differential equations. (15 mark)

$$\frac{dx}{dt} = 5x - 2y$$
$$\frac{dy}{dt} = x + 2y$$

b. The human malady of *ventricular arrhythmia* or *irregular heartbeat* is treated clinically using the drug *Lidocaine*. To be effective, the drug has to be maintained in the bloodstream and body tissue. The actual dosage depends upon body weight.
The following system of linear differential equations is described the concentration of *Lidocaine* in the bloodstream; $\dot{x}(t)$, concentration of *Lidocaine* in the body tissue; $\dot{y}(t)$, amount of *Lidocaine* in the bloodstream ; $x(t)$, and amount of *Lidocaine* in the body tissue; $y(t)$.

$$\dot{x} = -3x + \sqrt{2}y$$
$$\dot{y} = \sqrt{2}x - 2y$$

Solve the above system to calculate the amount of *Lidocaine* in the blood stream and amount of *Lidocaine* in the body tissue. (15 mark)

4.

a. In each of the following cases, find f_x, f_y, f_{xx}, f_{yy}, and f_{xy}, where f_x, f_y, f_{xx}, f_{yy}, and f_{xy} have usual meaning.

i. $f(x,y) = 4x^5 - 8xy^2 + 7y^5 - 3$ (05 mark)
ii. $f(x,y) = 4x^3y^2 \ln(3x^2y)$ (05 mark)
iii. $f(x,y) = e^{xy} \sin(2x^3y^2)$ (05 mark)

b. The height of a tree increases at a rate of 2 meters per year and the radius increases at 0.1 meters per year. What rate volume of timber increasing at when the height is 20 meters and the radius is 1.5 meters. (Assume the tree is a circular cylinder) (05 mark)

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