Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
300 Level 1st Semester Examination –Jul./Aug. 2016
MRT 305-3 Mathematical & Statistical Methods in Mineral Science

Uva Wellassa University

Instructions to candidates
Duration: Three(03) hours
Number of questions: Six(06) Essay Questions
Mark allocation: 200 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions

1.

a. Find a root, between 0 and 0.5, of the equation $4e^{-x}\sin x - 1 = 0$ to a tolerance 10^{-3}, by using **interval bisection method**. (10 mark)

b. Use **Newton-Raphson method** to find a root, with an accuracy 10^{-3} of the equation $2x - 3 = \cos x$ with $x_0 = 1.5$. (15 mark)

2.

a. Use **Cramer's rule** to solve the following system of linear equations. (10 mark)

$$\begin{aligned} 3x + 2y + 4z &= 7 \\ 2x + y + z &= 7 \\ x + 3y + 5z &= 2 \end{aligned}$$

b. An approximate solution of the system

$$\begin{aligned} 10x_1 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_1 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}$$

is given by

$$x_1^{(0)} = 0.3, x_2^{(0)} = 1.56, x_3^{(0)} = 2.886 \text{ and } x_4^{(0)} = -0.1368.$$

Use **Jacobi's method** to improve this solution. (15 mark)

4-4

3.

a. Determine the **Lagrange's interpolating polynomial** of degree 3, which fits the following data and hence find an approximate value of $y(10)$. (15 mark)

x	0	1	3	4
y	-12	0	12	24

b. Compute the **Newton's divided difference** table for the following points, hence find $f(x)$ as a polynomial in x. (10 mark)

x	$f(x)$
-1	3
0	-6
3	39
6	822
7	1611

c. Find the **linear splines** satisfying the following data. Determine the approximate values of $f(1.5)$ and $f(2.5)$. (10 mark)

x	1	2	3
$f(x)$	-8	-1	18

4.

a. The distances (x cm) traversed by a particle at different times (t seconds) are given below.

t	0.0	0.1	0.2	0.3	0.4
x	3.01	3.16	3.29	3.36	3.40

Use the most appropriate **three point formulas** to determine the velocity of a particle at $t = 0.0$, 0.2, and 0.4. (10 mark)

b. Evaluate

$$I = \int_0^1 \frac{1}{1+x}\,dx,$$

using **mid-ponit rule, trapezoidal rule** and **Simpson's rule** . (20 mark)

5.

a. Use **Euler's method** to find a numerical approximation for $x(t)$ where

$$\frac{dx}{dt} = f(x,t) = -2t^3 + 12t^2 - 20t + 8.5; x(0) = 1$$

from $t = 0$ to $t = 4$ using step size h =0.5. (20 mark)

b. Population growth of wild elephants is frequently modeled by an ordinary differential equation of the form

$$\frac{dN}{dt} = aN - bN^2, N(0) = N_0$$

where N is population size, aN represents birth rate, and bN^2 represents the death rate due to all causes, such as disease, competition for supplies, so on. If $N_0 = 100,000$, $a = 0.1$ and $b = 8 \times 10^{-7}$, calculate $N(t)$ for $t = 2$ with $h = 1$, using **Runge-Kutta fourth order formula**. (10 mark)

6.

a. Define and briefly discuss the following.

i. Geostatistics (05 mark)

ii. Random Variable (05 mark)

iii. Variogram (05 mark)

b. A researcher wishes to explain % Annual Population Growth (PopGrowth) by examining Infant Mortality Rate (IMR), Gross Domestics Product (GDP), Life Expectancy (LifeExpect), and % Rural Population (RuralPop). A random sample of 200 countries is used from the world bank databank for this study. Using the Minitab software, he generated the following incomplete Minitab output for the best-fitting multiple regression.

Output:

Table 01: Parameter Estimated

	Estimate	Std. Error	t value	Pr (>ItI)
(Intercept)	2.560e+00	1.934e+00	1.324	0.187251
GDP	2.973e-05	8.079e-06	3.680	0.000308
IMR	1.947e-02	7.932e-03	2.455	0.015019
LifeExpect	-3.063e-02	2.437e-02	-1.257	0.210451
RuralPop	4.395e-07	5.202e-03	0.000	0.999933

Multiple R-squared: 0.743, MSE: 12.9 and F_{cal}:45.6

Examine the above output and answer the following questions.

i. What is the fitted regression model? (05 mark)

ii. Does the model predict well and is it significant? Indicate how you reached your decision. (Construct the full ANOVA table with suitable hypothesis to answer this part). (15 mark)

iii. How much variation do these independent variables collectively explain (R^2)? *Comment on it.* (05 mark)

iv. Which independent variables, if any proved to be significant? (specify the null and alternative hypothesis with 5% level of significance) (10 mark)

v. What would your next step be in attempting to refine and further develop your model to explain % Annual Population Growth. (05 mark)

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