Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
100 Level 2nd Semester Examination – Jan./Feb. 2016
SCT 102-3 Calculus

Instructions to candidates
Duration: Three(03) hours
Number of questions: Six(06) essay questions
Mark allocation: 200 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions

1.

a. Evaluate the following limits.

i. $\lim\limits_{x\to 2}(x^2+3x-4)$ (03 mark)

ii. $\lim\limits_{x\to 0}\dfrac{\sqrt{5+x}-\sqrt{5}}{x}$ (05 mark)

iii. $\lim\limits_{x\to\infty}\dfrac{x^{15}+3x^{11}+6x^7+x^3+8}{4x^{15}+7x^{12}+5x^7+3}$ (05 mark)

iv. $\lim\limits_{x\to 2}\dfrac{x^{-3}-\frac{1}{8}}{x-2}$ (05 mark)

v. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{\sin 5x}$ (05 mark)

b. Find $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}$, using **Sandwich theorem.** (06 mark)

c. Let $f(x)=\begin{cases}1-2x, & x<0\\ 0, & x=0\\ 1+3x, & x>0\end{cases}$, then show that $\lim\limits_{x\to 0}f(x)$ exists and is equal to 1.

(05 mark)

d. Show that the function f, such that

$$f(x)=\begin{cases} x & \text{when } 0\le x<\frac{1}{2}\\ 1 & \text{when } x=\frac{1}{2}\\ 1-x & \text{when } \frac{1}{2}<x\le 1\end{cases}$$

has a discontinuity at $x=\frac{1}{2}$. (06 mark)

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2.

a. Use definition of derivative to prove $\frac{d(x)}{dx}=1$. (04 mark)

b. For each of the following y, find $\frac{dy}{dx}$.

i. $y=3x^4-5x^2+8x-7$ (03 mark)

ii. $y=\left(x^3+2x+1\right)^5$ (04 mark)

iii. $y=(3x+1)(5x^2+8)$ (05 mark)

iv. $y=\frac{x^2+1}{\sin x}$ (05 mark)

c. Let $f(x)=x^5-4x^3+2x+6$, then find $f'(1)$ and $f''(1)$. (04 mark)

d. Find $\frac{dy}{dx}$ given that $x^3+y^3-6xy=0$. (05 mark)

3.

a. Skecth the graph of $y=x^2+2x-3$ for $-\infty<x<+\infty$, using **first derivative test**. (15 mark)

b. Find the dimentions of the rectangle that has maximum area if its perimeter is 20cm. (05 mark)

c. A company estimates that its daily cost function (in suitable units) is $C(x)=x^3-6x^2+13x+5$ and its total revenue function is $R(x)=28x$. Find the value of x that maximizes daily profit.
(Hint: Profit function $P(x)$ = Revenue function $R(x)$ - Cost function $C(x)$) (10 mark)

4.

a. Integrate the following functions with repect to x.

i. $\int\left(3x^5-4x^3+\frac{7}{x^2}+1\right)dx$ (04 mark)

ii. $\int(2x+1)^{2016}dx$ (02 mark)

iii. $\int\frac{e^x-e^{-x}}{e^x+e^{-x}}dx$ (03 mark)

iv. $\int\cos(5x+1)dx$ (03 mark)

b. Use the substitution $u = \sin x$ to find $\int \sin^5 x \cos x dx$. (05 mark)

c. Compute the integral;

$I = \int x \sin x$, using **integration by parts**. (06 mark)

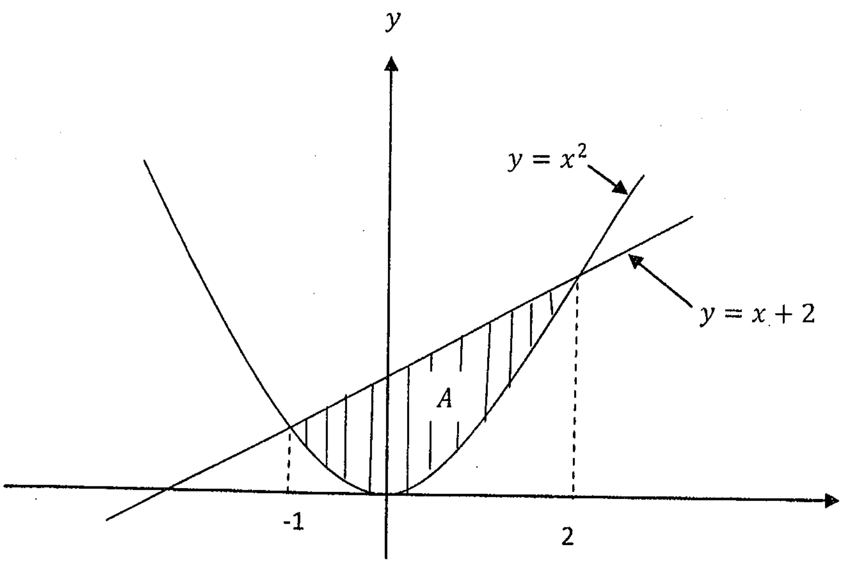
d. Evaluate the following definite integrals.

i. $\int_0^4 (x^3 - 6x + 1) dx$ (03 mark)

ii. $\int_2^5 \frac{du}{3u + 7}$ (03 mark)

e. Find the area of the region (A) in the figure 01 that is bounded by the curve $y = x^2$ and line $y = x + 2$.

Figure 01 : Graphs of $y = x^2$ and $y = x + 2$



(06 mark)

5.

a. Determine the following sequences are **convergent or divergent**:

i. $\left\{ \frac{n^2 + 5n - 4}{5n^2 - n + 7} \right\}$, $n \in \mathrm{N}$ (05 mark)

ii. $\{2n - 1\}$, $n \in \mathrm{N}$ (05 mark)

iii. $\{5 + (-1)^n\}$, $n \in \mathrm{N}$ (05 mark)

b. Determine whether each of the following series **converges or diverges**:

i. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\ldots\ldots\ldots$ (05 mark)

ii. $1+2+4+8+\ldots\ldots\ldots\ldots\ldots$ (05 mark)

6.

a. Use **L' Hospital's rule** to find the following limits:

i. $\lim_{x\to 1}\frac{x^3-2x^2+4x-3}{4x^2-5x+1}$ (04 mark)

ii. $\lim_{x\to 0}\frac{x-\sin x}{x^2}$ (05 mark)

b. Verify that the conditions of **Rolle's theorem** are satisfied by the function:

$$f(x)=x^4-2x^2, \quad x\in[-2,2]$$

and determine whether a value of c in $(-2,\ 2)$ for which $f'(c)=0$. (06 mark)

c. State the **mean value theorem**. (02 mark)

d. Verify that the conditions of the **mean value theorem** are satisfied by the function $f(x)=2x^2-7x+10$ in $[2,5]$; and find a value for c that satisfies the conclussion of the theorem. (06 mark)

e. Determine which of the following are propositions. If a given statement is proposition, then write down its truth value.

i.	P	: "3 is a prime number"	(01 mark)
ii.	Q	: "$x+1=2$"	(01 mark)
iii.	R	: " Read this question carefully"	(01 mark)

f. Determine the truth value of the following statements.

i. " 2 is a prime number" or " $2+2=5$". (01 mark)

ii. " Blue is a colour" and " $5^0=0$". (01 mark)

iii. If " one year= 12 months" then " $5+6=10$". (01 mark)

iv. If " $5>4$" onlyif "$4<3$". (01 mark)

g. Construct the truth table for the statement $P\vee(\sim q)\to\sim P$. (10 mark)