Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Computer Science and Technology
200 Level 2nd Semester Examination – Dec. / Jan. 2017/18
CST 213-2 Discrete Mathematics II

Uva Wellassa University

Instructions to candidates
Duration: Two (02) hours
Number of questions: Four (04) essay questions
Mark allocation: 100 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions.

1.

a. Consider the initial value problem as given below.

$$y' = 2y^2 + xy^2 \quad , y(0) = 1$$

i. Find the solution for above initial value problem. (05 mark)

ii. Hence, show that, $\dfrac{dy}{dx} = \dfrac{8+4x}{(2-4x-x^2)^2}$ (02 mark)

iii. Thus, determine where the solution attains its minimum value. (03 mark)

b. Consider the initial value problem as gven below.

$$y' + \frac{1}{4}y = 3 + 2\cos 2t \qquad , y(0) = 0$$

i. Find the **Integrating factor** for above initial value problem. (02 mark)

ii. Find the solution for above initial value problem and describe its behaviour for large ' t '. (06 mark)

iii. Determine the value of y when $t = 1.5$. (02 mark)

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c. Answer following questions considering $f(x)$ as given below.

$$f(x)=\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$$

(02 mark)

i. Find the solution for , $f(x)=0$.

(04 mark)

ii. Hence, solve $f(x)=-5+6x$.

iii. If $\frac{1}{f(D)}e^{ax}=\frac{1}{f(a)}e^{ax}$,

then show that solution of $f(x)=e^{5x}$ is $f(x)=\frac{e^{5x}}{6}+Ae^{2x}+Be^{3x}$, where A and B are arbitary constants.

(04 mark)

2.

(06 mark)

a. Solve the following system of equations .

$$\frac{dx_1}{dt}=3x_1-2x_2$$

$$\frac{dx_2}{dt}=2x_1-2x_2$$

b.

(02 mark)

i. Define partial differential equations.

ii. Write the general form of Quasi Linear First Order partial differential equations.

(02 mark)

iii. Solve the following equation.

(05 mark)

$$(y+u)u_x-(x+u)u_y+(x-y)=0$$

3.

(04 mark)

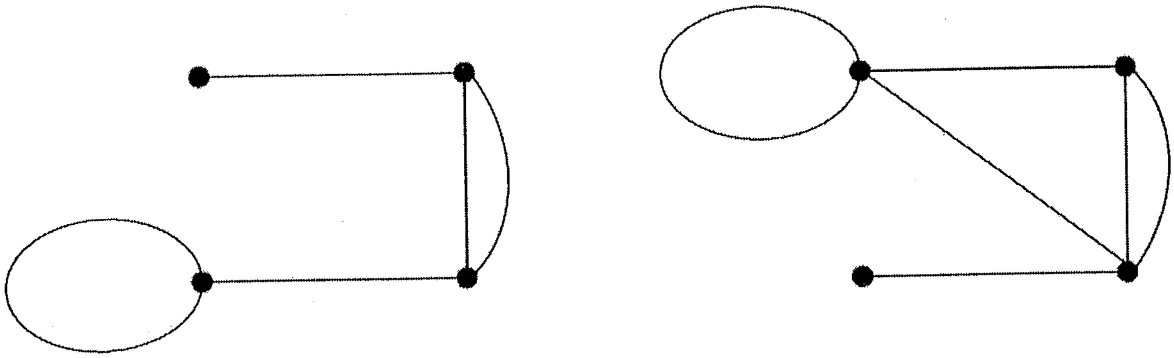
a. Draw the graph represented by the given adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 3 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

b. Determine whether the following pairs of graphs are *isomorpic* or *not*. **Justify your answer**.

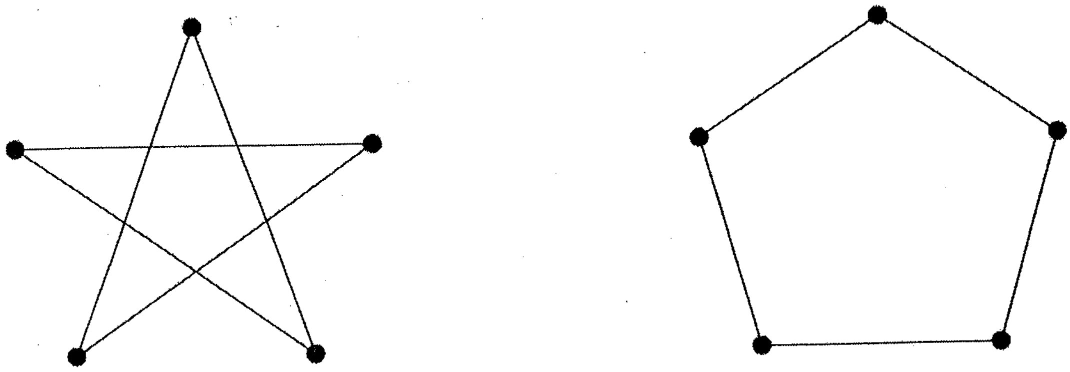
(03 mark)

i.



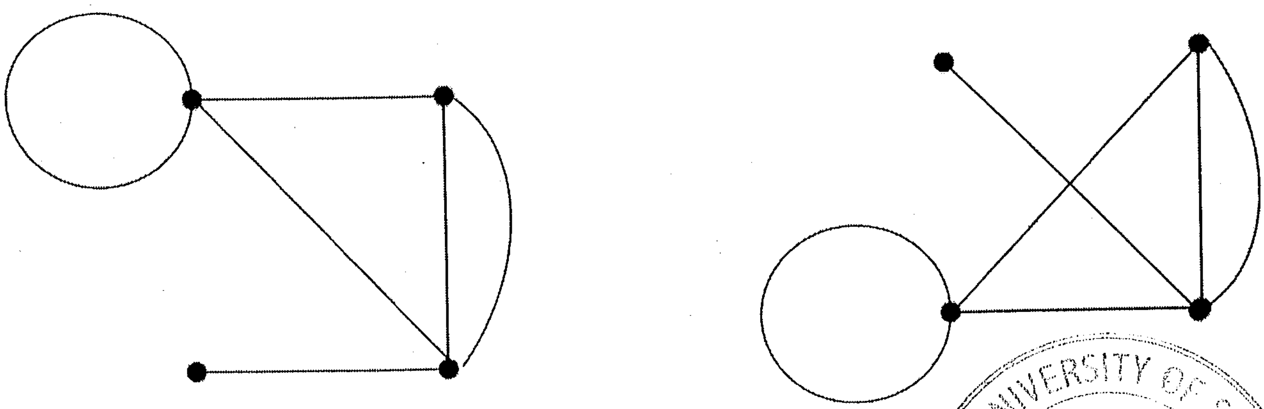
(03 mark)

ii.

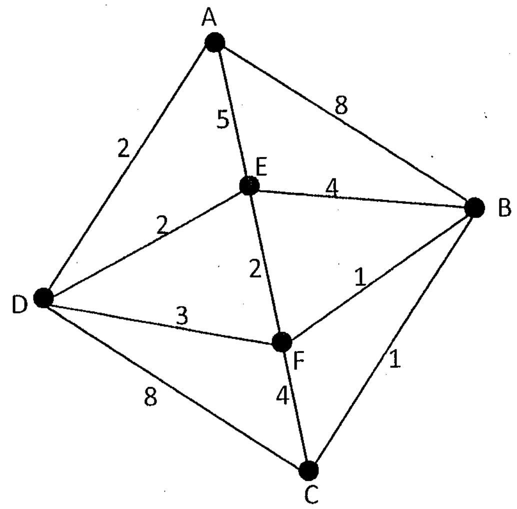


(03 mark)

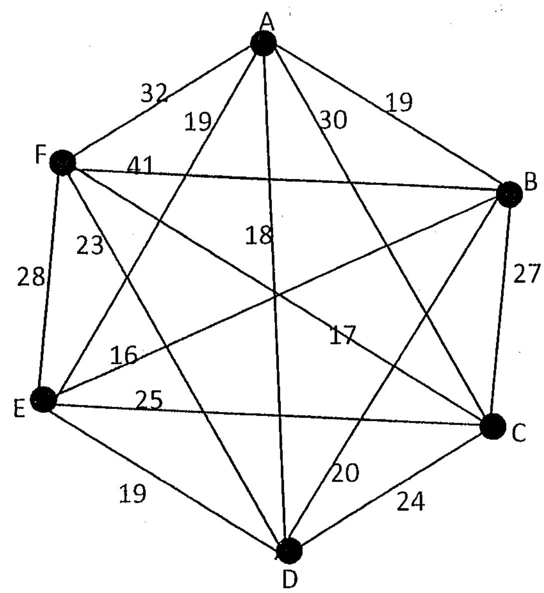
iii.



c. A postman wishes to deliver letters every day in a network of streets, covering the least possible total distance and return to the post office. He must travel each road in his route, at least one, but should avoid covering too many roads more than once. Find the minimum distance he has to travel.
(Where a non-negative numbers associated with each edge that represents the actual distance between corresponding nodes.) (06 mark)

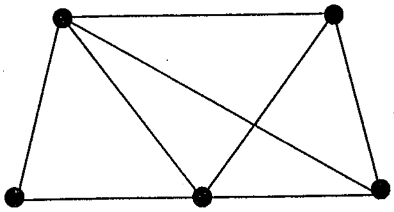


d. The following weighted graph represents a network of roads connecting a collection of towns. A travelling salesperson has to visit each town and return to his or her staring point. It is required to choose a route which minimizes the total distance of the round trip. Find a shortest possible round-trip route. (06 mark)

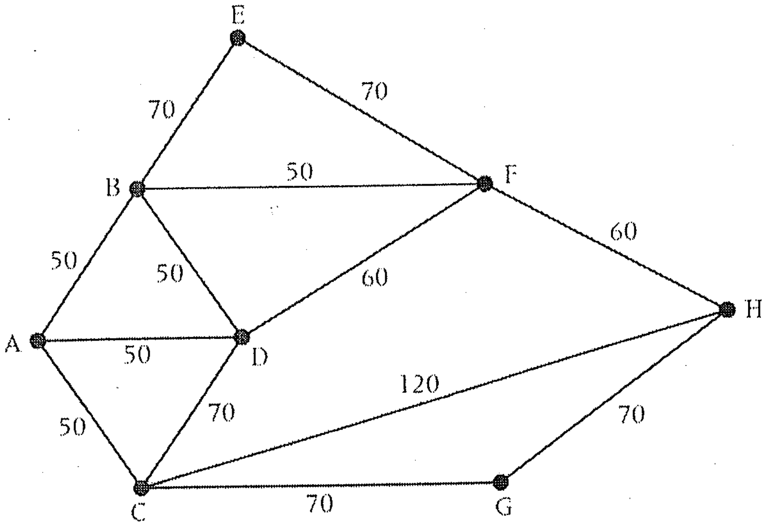


a. Define **Spanning Tree.** (02 mark)

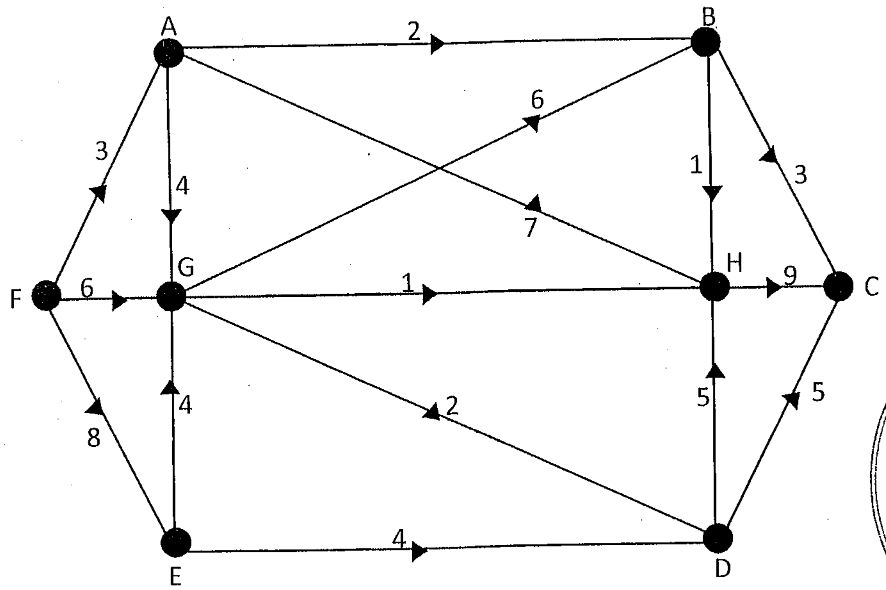
b. Draw all possible spanning trees of following graph. (05 mark)



c. Use **Prim's algorithm** or **Kruskal's algorithm** to find a *minimum spanning tree* for the given weighted graph. (05 mark)



d. Find the maximum flow for the following network. Find a cut with capacity equal to this maximal flow. Hence, verify the **maximum flow using minimum cut theorem.** (09 mark)



e. Suppose that a chemist wishes to store 6 chemicals a, b , c , d , e and f in various areas of a ware house. Some of these chemicals react violently when in contact and so must kept apart (in separate areas). Following table illustrates reactions of chemicals. Find the minimum number of areas required to store 6 chemicals. (09 mark)

	a	b	c	d	e	f
a	-	*	*	*	-	-
b	*	-	*	*	*	-
c	*	*	-	*	-	*
d	*	*	*	-	*	*
e	-	*	-	*	-	*
f	-	-	*	*	*	-

(chemicals which react together are indicated by *)