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Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Science and Technology Degree Programme
1st Year 2nd Semester Examination – August/September 2014

Uva Wellassa University

SCT 102-2 Mathematics I

Number of questions: Four (04)
Answer all questions
Time allocation: Two (02) hours
Total marks: 100

1. a. Obtain the first four terms of each of the following sequences.

(i) $\left\{\frac{\sqrt{n}}{n+1}\right\}$, (ii) $\left\{\frac{(2x)^{n-1})}{(2n-1)^5}\right\}$, (iii) $\left\{\frac{(-1)^{n-1}}{2.4.6...2n}\right\}$, (iv) $\left\{\frac{(-1)^{n-1}x^{2n-1}}{(2n-1)!}\right\}$

(10 mark)

b. (i) What is the "limit of an infinite sequence"?

(5 mark)

(ii) Using the theorems of limit, evaluate the limit of each of the following sequence.

$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$, $\lim_{n\to\infty}\left\{\frac{n(n+2)}{n+1}-\frac{n^3}{n^2+1}\right\}$

(10 mark)

2. a. Two students, A and B, were asked to write an *n*th term of the sequence 1, 16, 81, 256,.....and to write the 5th term of the sequence. A gave the *n*th term as $u_n = n^4$. B wrote $u_n = 10n^3 - 35n^2 + 50n - 24$. Which student gave the correct 5th term?

(10 mark)

b. A sequence has the *n*th term given by $u_n = \frac{3n-1}{4n+5}$

(i) Write the 1st, 5th, 10th, 100th, 1000th, 10,000th and 100, 000th terms of the sequence in decimal form. Make a *guess* as to the limit of this sequence as $n \to \infty$.

(5 mark)

(ii) Using the definition of limit, verify that the *guess* in b (i) is actually correct.

(10 mark)

3. a. (i) Define an *infinite series.* What do you mean by *convergence* of an infinite series?

(5 mark)

(ii). Prove that the series $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} \ldots\ldots + \ldots =$ $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+3)}$ converges. Find its sum.

(5 mark)

(iii). Investigate whether the series $\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \ldots\ldots =$ $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ converges. Find the sum.

(5 mark)

b. i. Write the integral test for *convergence* and *divergence* for series of non-negative terms.

(5 mark)

ii. Investigate the convergence of

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}, \quad \sum_{n=1}^{\infty} n e^{-n^2}$$

(5 mark)

4. a. If f is a function of x, y *and* z, i.e., $f = f(x, y, z)$, then define the partial derivative of f with respect to x *and* y.

(5 mark)

b. What are "higher order partial derivatives"?

(5 mark)

c. If $f\ (x, y. z) = x^3yz^2 + e^{x^2 2y^3 3z}$, find

f_x , f_y , f_z, f_{xx}, f_{yy}, f_{zz}

(15 mark)