UvaWellassa University, Sri Lanka
Faculty of Management
Bachelor of Business and Management in Insurance and Actuary
1st Semester Examination – March/ April 2013

Uva Wellassa University

EMG 311-0 Essential Mathematics for Actuarial Science

Instructions to candidates
Number of questions: Five (04)
Answer all questions
Time allocation: Two (02) hours
Total marks allocated: 100
Calculators are allowed

1.

a. The function $f : R \rightarrow R$ is defined by $f(x) = 2x^2 + 2x + 3$. Find

i. $f(2)$

ii. $f(t-1)$, where t is a constant

b. If $f(x) = \frac{(x-1)}{x}$, then show that $f(x)f(1-x) = f(-1)$

c. $f(x) = x^2 + 1$ and $g(x) = 2x + 3$ are given.

i. Find $fog(x)$

ii. Compute $fog(2)$

d. Find the domain and the range of the following function

i. $f(x) = 2x^2 + x - 1$

ii. $f(x) = \sqrt{x-16}$

iii. $f(x) = |x+1|$

(25 marks)

12

2.

a. Evaluate each of the following limit

i. $\lim_{x\to 2}\left(x^3+2x^2+x-2\right)$

ii. $\lim_{x\to 1}\left(\frac{x^3-2}{x+1}\right)$

iii. $\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)$

iv. $\lim_{x\to 0}\left(\frac{3x^3-2x^2+2x}{4x^2+3x}\right)$

v. $\lim_{x\to \infty}\left(\frac{3x^4+3x^3+2x^2-x+1}{2x^4+3x^3-2x+3}\right)$

b. The function $f(x)$ is defined by,

$$f(x)=\begin{cases} x+2 & if \quad 0\le x\le 1 \\ 4x-1 & if \quad 1<x\le 2 \end{cases}$$

i. Sketch the graph of $f(x)$

ii. Evaluate the limit $\lim_{x\to 1} f(x)$

iii. Show that the function is continuous at $x=1$

c. It is estimated that t years from now the population of a certain suburban community will be $P(t)=20-\frac{6}{t-1}$ thousand.

The population will increase and reach to a certain fixed level over long period. What will be that value? Justify your answer using limit concept.

(25 marks)

3.

a. Find the derivative of $f(x)$ with respect to x

i. $f(x)=3x^3+2x^2+3x-2$

ii. $f(x)=\frac{1}{4}x^4+\frac{1}{2}x^2+3x^{1/2}$

iii. $f(x)=5e^x+3x-\pi$, where e is the exponential number

iv. $f(x)=(2x+1)(x-2)$

b. $y = f(u) = 2u^3$ and $u = g(x) = x^2 + 2x - 2$ are given. Use chain rule to find the derivative of y with respect to x (i.e., $\frac{dy}{dx}$)

c. A company produces DVD-drivers. The cost of producing q number of DVD-drivers is given by $C(q) = 1000 + 5q + 3q^2$ in rupees.

i. Find the derivative of C with respect q.

ii. What is the marginal cost of manufacturing the 51st driver?

(25marks)

4.

a. The function $y = f(x) = 2x^3 - 12x^2 + 18x + 1$ is given.

i. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

ii. Find the critical point of $f(x)$ and use second derivative test to determine whether they are maximums or minimums

iii. Compute their maximum and minimum values

b. Find the following integrals

i. $\int (3x^2 + 4x - 2x + 3)dx$

ii. $\int x(x+2)dx$

c. Evaluate the definite integral $\int_0^2 (6x^2 + 4x + 1)dx$

d. ABC Fab installs a machine, which dispenses hot drinks. The marginal revenue $R'(t)$ is a function of time. Also, the cost of servicing the machine, function $C'(t)$, increase as time passes, since parts wear out and need to be replaced. The marginal revenue and the cost functions are $R'(t) = 200 - 2t^2$ and $C'(t) = 25 + 2t + t^2$ respectively, t is in years.

i. Find the revenue function $R(t)$ and the cost function $C(t)$

ii. Compute the profit from using the machine for 4 years.

(25marks)