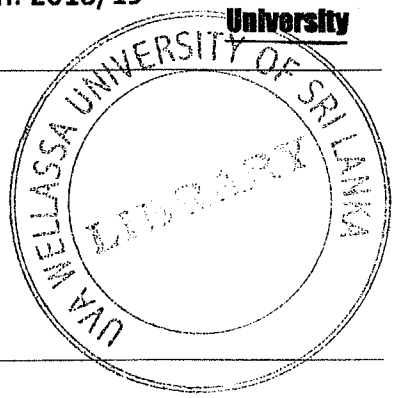
Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Computer Science and Technology
300 Level 2nd Semester Examination – Dec./Jan. 2018/19
IIT 311-3 Operational Research



Instructions to candidates
Duration: Three (03) hours
Number of questions: Six (06) Essay Questions
Mark allocation: 150 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions

1. Janith is a commercial farmer. He is going to plant *corn* and *green gram*. Each acre planted with corn yields Rs. 200,000 profit and each with green gram yields Rs. 300,000 profit. The labor and fertilizer used for each acre given in the following table. One hundred workers and 120 tons of fertilizer are available. His objective is to maximize the total profit of the session.

	Corn	Green gram
Labor (per acre)	3 workers	2 workers
Fertilizer (per acre)	2 tons	4 tons

   a. Formulate a mathematical model for this scenario. (05 mark)
   b. Use the **graphical method** to solve the model formulated in part (a). (07 mark)
   c. How many acres of each crop should be cultivated to yield maximum profit? (03 mark)

2. A firm makes two types of containers, A and B, each of which requires cutting, assembly and finishing. The maximum available machine capacity in hours per week for each process is: cutting 50, assembly 84, and finishing 72. The processing times for one unit of each type are as follows:

Process	Time in hours	
	A	B
Cutting	2	5
Assembling	4	8
Finishing	4	5

The profit margin is Rs. 600 per unit A and Rs. 1000 per unit B. Manager's objective is to maximize the weekly profit.

   a. Formulate this as a linear programming problem. (05 mark)
   b. Determine the optimum weekly output of containers and maximum profit by using the **simplex method**. (20 mark)

3. A distribution Agency has three factories in the three cities viz. A, B, and C. These three factories supply consignments to three dealers viz. X, Y, and Z. The dealers are spread all over the country. Weekly factory capacities of A, B, and C factories, weekly dealer requirements and unit transportation costs (in Rupees) are given in following table.

From \ To	X	Y	Z	Supply
A	6	7	9	100
B	5	3	2	180
C	8	5	7	200
Demand	135	175	170	

The general administration manager wants to determine the best plan for how many shipments to send from each factory to the respective dealers in each week. The manager's objective is to minimize the total transportation cost.

a. Formulate a mathematical model for this problem. (05 mark)

b. Use the **north-west corner rule** to obtain an initial basic feasible solution for the model formulated in part (a). (06 mark)

c. Discuss the degeneracy of the solution in part (b). (02 mark)

d. Starting with the initial basic feasible solution from part (b), find the optimal solution to this problem. (12 mark)

4. Human Resource Division of the Uva Wellassa University has four jobs to assign to four newly recruited workers. The estimated costs (in thousands of rupees) of assigning a particular worker to a particular job are shown in the table below.

Workers \ Jobs	Office Assistant	Book Keeper	Computer Operator	Lab Labor
A	5	7	11	6
B	8	5	9	6
C	4	7	10	7
D	10	4	8	3

The objective is to assign workers to jobs such that total assignment cost is a minimum. Only one worker can work on any one job.

a. Formulate a mathematical model for this scenario. (05 mark)

b. Use the **Hungarian algorithm** to obtain an optimal assignment and find the minimum total assignment cost. (15 mark)

5. *Kingswood Park* has recently been set aside for a limited amount of sightseeing and backpack hiking. Personal vehicles are not allowed into the park, but there is a narrow, winding road system for trams and for jeeps driven by the park rangers. This road system is shown (without the curves) below, where location "*O*" is the entrance into the park; other letters designate the locations of ranger stations (and other limited facilities). The numbers indicate the distances of these winding roads in miles. The park contains a scenic wonder at station "T". A small number of trams are used to transport sightseers from the park entrance "O" to station "T" and back. The park management currently faces four problems. Therefore, management has decided to recruit a BIIT graduate from the Uva Wellassa University to solve the following problems. Imagine that you are selected to do this job.

a. One problem is to determine which route from the park entrance "O" to station "T" has the smallest total distance (shortest path) for the operation of the trams. Then, you should determine the *shortest path* from the park entrance "O" to station "T" using ***Dijkstra's Algorithm*** (Use figure 5.1). (10 mark)

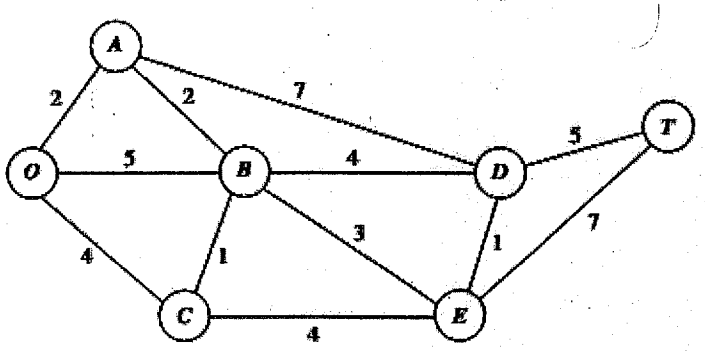


Figure 5.1

b. A second problem is that telephone lines must be installed under the roads to establish telephone communication among all the stations (including the park entrance). Because the installation is both expensive and disruptive to the natural environment, lines will be installed under just enough roads to provide some connection between every pair of stations. The question is where the lines should be laid to accomplish this with a minimum total number of miles of line installed. Then, you should determine the *minimum spanning tree* of the given road network using ***Kruskal's Algorithm*** or ***Prim's algorithm*** (Use figure 5.1). (10 mark)

c. The third problem is that more people want to take the tram ride from the park entrance "O" to station "T" that can be accommodated during the peak season. To avoid unduly disturbing the ecology and wildlife of the region, a strict ration has been placed on the number of tram

trips that can be made on each of the roads per day. (These limits differ for the different roads) Therefore, during the peak season, various routes might be followed regardless of distance to increase the number of tram trips that can be made each day. The question pertains to how to route the various trips to maximize the number of trips that can be made per day without violating the limits on any individual road. Then, you should find the *maximum flow* from the park entrance "O" to station "T" using ***Augment Path algorithm***. (Use figure 5.2) (06 mark)

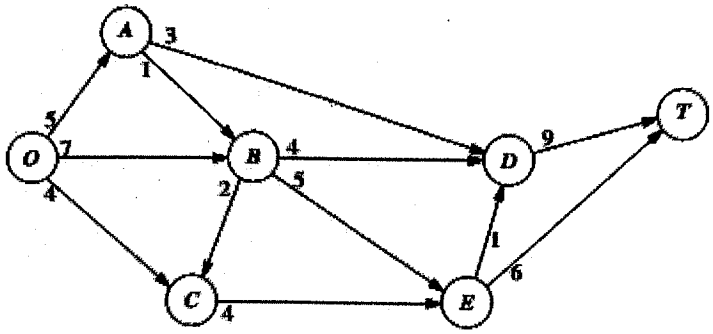


Figure 5.2

d. The fourth problem is that the government wants to construct an alternative road from area X to area Y through the park. However, the management has informed that the proposed road should be constructed as reducing the damage to the existing road network and natural beauty. Then, you should propose the optimal method to construct that new road to connect area X and area Y. (Hint: *minimum cut* – use figure 5.3) (04 mark)

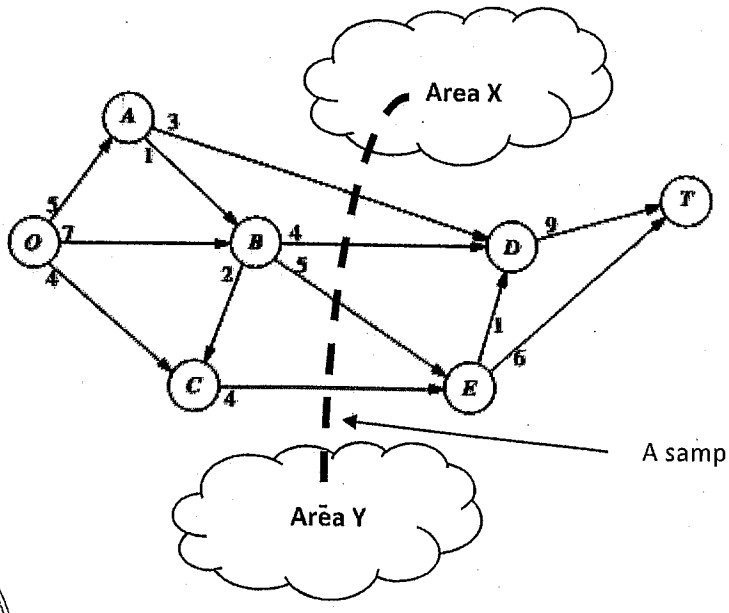


Figure 5.3

6.

a. Mrs. Amanda, Vice President of marketing for the *Mario Electronic Toys Company*, is about to begin a project to design an advertising campaign for a new line of toys. She is required to complete the entire project within twenty (20) days in time to launch the advertising campaign at the beginning of the next Christmas season. She has identified the eleven (11) activities (labeled as A, B,..., K) needed to execute this project. Considering the order in which these activities need to occur, she also has constructed the following table.

Activity	Predecessor Activity	Time estimate (In days)
A	-	5
B	-	6
C	-	3
D	A	5
E	B	7
F	B	10
G	C	4
H	D	2
I	E	5
J	F, G	6
K	H, I, J	4

i. Draw the **Critical Path Method (CPM) project network (activity-on-node)** for this project. (08 mark)

ii. Calculate the *earliest and latest start and finish times, the slack for each activity, and the critical activities*. (05 mark)

iii. What is the least amount of time required to complete the project? Can Mrs. Amanda complete the project within twenty (20) days? (02 mark)

b. Prof. A.B.C Silva is the President of the Research Division of Better Health, Inc., a major pharmaceutical company. His most important project coming up is the development of a new drug to combat HIV AIDS. He has identified 9 groups in his division, which will need to carry out the different phases of this research and development project. Referring to the work to be done by the respective groups as activities A, B, C, D, E, F, G, H, and I. To beat the competition, Better Health's CEO has informed Professor that he wants the drug ready within 50 days. The professor knows very well that there is considerable uncertainty about how long each group will need to do its work. The leader of each group has provided a most likely estimate; an optimistic estimate, and a pessimistic estimate of the time duration of that group's activity. The precedence relationships for when these groups need to do their work and Program Evolution & Review Techniques (PERT) three estimates (in days) are shown in the following table.

Activity	Predecessor Activity	Optimistic time estimate (a)	Most likely time estimate (m)	Pessimistic time estimate (b)
A	-	2	4	6
B	A	3	6	9
C	A	8	10	12
D	B	9	12	15
E	C	8	9	10
F	D, E	16	21	26
G	D, E	19	22	25
H	F	2	5	8
I	G	1	3	5

i. Compute the expected activity time for each activity using PERT formula (round to the nearest integer). (04 mark)

$$\text{Expected time } (t_i) = \frac{a + 4m + b}{6}$$

ii. Draw the **PERT project network (activity-on-node)** for this problem. (08 mark)

iii. Calculate the *earliest and latest start and finish times*, the *slack for each activity*, and the *critical activities*. (05 mark)

iv. What is the least amount of time required to complete the project? (01 mark)

v. What should Professor tell his CEO about the likelihood that the drug will be ready within 50 days? (02 mark)

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