

Instructions to candidates

Duration: Two(02) hours

Number of questions: Four(04) Essay Questions

Mark allocation: 100 mark

Use standard symbols without definition.

Scientific calculators are allowed.

Answer all questions

1. Find each of the following limits.

i. $\lim_{x \rightarrow 2} (3x - 2)$ (03 mark)

ii. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$ (03 mark)

iii. $\lim_{x \rightarrow \infty} \frac{x^4 + 3x^3 - 6x + 10}{4x^4 - x^2 + x + 7}$ (04 mark)

iv. $\lim_{x \rightarrow -3} \frac{x + 3}{\sqrt{x + 7} - 2}$ (05 mark)

v. $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x^3 - 8}$ (05 mark)

vi. $\lim_{x \rightarrow 0} \frac{\sin 6x}{\sin 4x}$ (05 mark)

2.

a. Use definition of derivative to prove $\frac{d(x^2)}{dx} = 2x$. (05 mark)

b. Differentiate following functions with respect to x .

i. $y = 4x^{15} + 6x^{13} - 2x^7 + 3x^5 + 2x^2 + 15$ (04 mark)

ii. $y = (4x - 3)(x^2 + 1)^2$ (04 mark)

iii. $y = e^{3x} \ln 2x$ (04 mark)

iv. $y = \frac{3x^3 - 2x^2 - 1}{x^3 + 1}$ (04 mark)

v. $y = \ln(\tan 3x)$ (04 mark)



3.

a. Draw the graph of $y = x^2 + 2x - 3$, by using **first derivative test**. (10 mark)

b. The length of a rectangular block of wood is twice its width. The total surface area is 108 cm^2 . If the width of the block is $x \text{ cm}$, show that its volume is $\frac{4}{3}x(27 - x^2) \text{ cm}^3$. Find the dimensions of the block when its volume is a maximum. Justify your answer. (15 mark)

4.

a. Integrate the following functions with respect to x .

i. $\int (6x^{2016} + 4x^{2005} + 2x^{1995} + 1985) dx$ (04 mark)

ii. $\int \left(\frac{2x - 5x^{1/2}}{x^2} \right) dx$ (04 mark)

iii. $\int \frac{x + 2}{x^2 + 4x - 7} dx$ (03 mark)

iv. $\int \sin 3x \cos 3x dx$ (04 mark)

b. Find the value of the following integrals.

i. $\int_0^4 (2 - \sqrt{x}) dx$ (05 mark)

ii. $\int_1^e x \ln x dx$ (05 mark)
