Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
100 level 2nd Semester Examination – Dec. / Jan. 2017/18
SCT102-3 Calculus

Uva Wellassa University

Instructions to candidates
Duration: Three (03) hours
Number of questions: Six (06) Essay questions
Mark allocation: 200 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions.

1.

a. Estimate the limit of, $\lim_{x \to 0}\left(\frac{x}{\sqrt{x+1}-1}\right)$ numerically using a table. (04 mark)

b. Evaluate the following limits,

i. $\lim_{x \to 1}(7x+1)$ (02 mark)

ii. $\lim_{x \to 1}\left(\frac{\sqrt{x}-1}{x-1}\right)$ (03 mark)

iii. $\lim_{x \to -2}\left(\frac{x^3+8}{x+2}\right)$ (03 mark)

iv. $\lim_{x \to 4}\left(\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}\right)$ (03 mark)

v. $\lim_{x \to \infty}\left(\frac{125x^3+56x^2+x+45}{5x^3-13x^2+51x-63}\right)$ (03 mark)

c. State the **Sandwich theorem** and hence compute, $\lim_{x \to \infty}\left(\frac{\sin x}{x}\right)$.

(05 mark)

d. Let $f(x)=\begin{cases} 0 & ; \ if \quad x \le 0 \\ x^2 & ; \ if \quad 0 < x \le 1 \\ 2x & ; \ if \quad x > 1 \end{cases}$

i. State whether $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$ exist or not. Justify your answer.

(07 mark)

ii. Show that the function $f(x)$ is continuous at $x = 0$ and function $f(x)$ is discontinuous at $x = 1$. (05 mark)

10

2.

a. Use the ***definition of derivative*** to prove $\frac{d}{dx}(3x^2) = 6x$. (06 mark)

b. Find the derivative of following functions with respective x.

i. $y = 8x^5 + 9x - x^{3/2} + \sqrt{x}$ (02 mark)

ii. $y = (x^3 + 8)(x^2 - 3)$ (03 mark)

iii. $y = \frac{\ln(x)}{\sqrt{x}}$ (03 mark)

iv. $y = (e^{2x}\sin x)^7$ (04 mark)

v. $y = \sqrt{\frac{1-\sin x}{1+\sin x}}$ (04 mark)

c. Let $f(x) = x^4 + 6x^3 - x + \sqrt{2}$, then find $f'(x)$, $f''(x)$and $f'''(x)$. (03 mark)

d. Let $f(x) = 2 - x + x^2$,

i. Find the interval for x when $f(x)$ is increasing or decreasing. (05 mark)

ii. Find the value of local minimum or local maximum. (01 mark)

iii. Find the point of inflextion. (02 mark)

iv. Sketch the graph of $f(x)$. (05 mark)

e. A rectangular playground of a university is shown in figure 01. The dimensions in meters of the playground are indicated. The perimeter of the playground is 100 m.

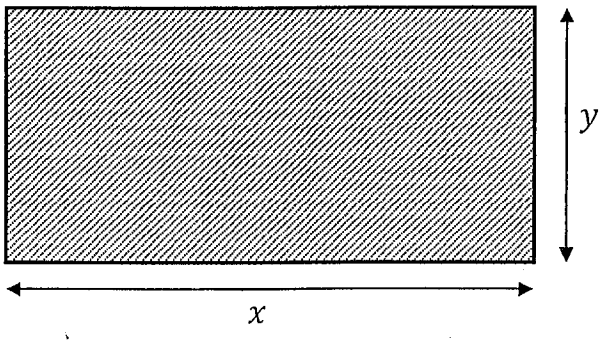

Figure 01 : Playground

I. Express y in terms of x. (01 mark)

II. Show that the area (A) of the playground, is $A = 50x - x^2$. (02 mark)

III. Hence, find the value of length (x) and width(y) of the playground by maximizing A. (04 mark)

3.

a. Evaluate following integrals with repective to x.

i. $\int (4x^3 + \sqrt{x})dx$ (02 mark)

ii. $\int (\sin 3x)dx$ (03 mark)

iii. $\int xe^{6x}dx$ (03 mark)

b. Evaluate the integral $\int \frac{4x}{\sqrt{2x^2+1}}dx$ by **substitution** $u = 2x^2 + 1$.

(05 mark)

c. Use **Integration by parts** to show, $\int (3t+5)\cos\left(\frac{t}{4}\right)dt = 4(3t+5)\sin\left(\frac{t}{4}\right) + 48\cos\left(\frac{t}{4}\right) + c$,

where c is an arbitrary constant.

(05 mark)

d. Find the values of constants A, B and , such that $\frac{x^2+4}{x(x+2)(3x-2)} = \frac{A}{x} + \frac{B}{(x+2)} + \frac{C}{(3x-2)}$.

Hence, compute the integral $\int_1^2 \frac{x^2+4}{x(x+2)(3x-2)}dx$.

(05 mark)

e. Find the shaded area in figure 02, that is bounded by the curves $y = 0.25x^2 + 3$ and $y = -x^2 + 8$. (07 mark)

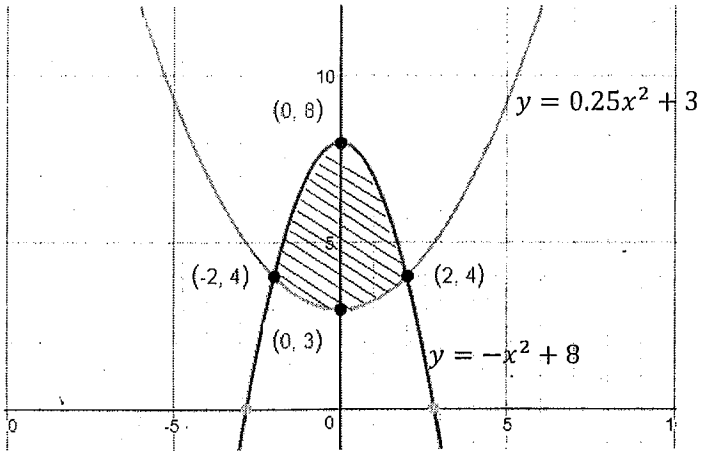

Figure 02: Graphs of $y = 0.25x^2 + 3$ and $y = -x^2 + 8$

4.

a. Obtain the first four terms of the following sequences.

i. $\left(\frac{n}{n+1}\right), n \in \mathbb{N}$ (04 mark)

ii. $\left(\frac{(-1)^n}{n}\right), n \in \mathbb{N}$ (04 mark)

b. Determine whether the following sequences are *convergent* or *divergent*. **Justify your answer**.

i. $(3), \ n \in \mathbb{N}$ (02 mark)

ii. $\left(\frac{1}{n^2}\right), n \in \mathbb{N}$ (03 mark)

iii. $\left(\frac{3n^2+8n+3}{n^2+5}\right), n \in \mathbb{N}$ (04 mark)

iii. $\left(1+(-1)^n\right), n \in \mathbb{N}$ (04 mark)

c. Determine whether the following series are *convergent* or *divergent*. **Justify your answer.**

i. $\sum_{n=1}^{\infty} 2^n$ (02 mark)

ii. $\sum_{n=1}^{\infty}\left(\frac{1}{n(n+1)}\right)$ (03 mark)

iii. $\sum_{n=1}^{\infty}\left(\frac{n^2+n+1}{3n^2-5n+2}\right)$ (03 mark)

d. Find the **Taylor series** of the function $f(x) = e^{-x}$ about $x = 0$. (05 mark)

5.

a. Use the **L'Hospital's rule** to find the following limits.

i. $\lim_{x \to 0}\left(\frac{\sin x}{x}\right)$ (03 mark)

ii. $\lim_{x \to \infty}\left(\frac{e^x}{x^2}\right)$ (04 mark)

iii. $\lim_{x \to 0}\left(\frac{e^{3x}-1-3x}{e^{x^2}-\cos x}\right)$ (04 mark)

b. State the **Rolle's Theorem** and verify the Rolle's Theorem for the function $f: [-1,2] \to \mathbb{R}$, $f(x) = (x+1)^5(x-2)^7$ on the interval $[-1,2]$.

(06 mark)

c. State **Mean Value Theorem** and hence prove that $\frac{\pi}{4} - \frac{5}{41} < \tan^{-1}(0.8) < \frac{\pi}{4} - \frac{1}{10}$.

(10 mark)

6.

a. Which of the following sentences are *propositions*. If a given statement is a proposition, then write down its *truth value*.

i. Today is a rainy day. (02 mark)

ii. $8 < 3$ (02 mark)

iii. May God bless you (02 mark)

iv. π is rational number (02 mark)

b. What is the *negation* of each of the following propositions?

i. 1 is a prime number. (02 mark)

ii. $5 + 4 = 9$ (02 mark)

iii. $4 < 9$ (02 mark)

c. Consider the following statements,

P: e is irrational
Q: 15 is not a triangular number

Find the truth value of the following propositions based on above information.

i. $\sim P$ (02 mark)

ii. $P \wedge Q$ (02 mark)

iii. $P \vee Q$ (02 mark)

iv. $P \Rightarrow Q$ (02 mark)

v. $P \Leftrightarrow Q$ (02 mark)

d. Construct the *truth table* for the statement $P \Rightarrow [\sim(Q \vee R)]$.

(05 mark)