Uva Wellassa University, Sri Lanka
B.Tech. Degree Programme - 2007/08
End Semester Examination- Semester 1
January -2008

MAT 101-2 Essential Mathematics

Answer Any Four (4) Questions Time: Two (02) Hours

(1) (*i*) State whether following statements are true or false.

(a) $0.1 \in \mathbb{N}$ (b) $\mathbb{N} \subset \mathbb{R}$ (c) $3.3 \notin \mathbb{R}$ (d) $0 \in \mathbb{Z}$

(*ii*) The area A of any circle depends on its radius r. f is a function that gives an area of any circle with radius r as follows,

$$A = f(r)$$
$$f(r) = \pi r^2$$

(a) Find the domain of f.

(b) Find the area of a circle with radius $1cm$.

(c) Find the area of a circle with radius $2cm$.

(d) Is the area of 2 cm radius circle is larger than that of with radius 1 cm ?

(2) (*i*) (a) Convert 30 degrees into radians.

(b) Convert π radians into degrees.

(*ii*) Replacing θ of following equations find the relation between x & y.

$$y = 3cos\theta$$
$$x = 4sin\theta$$

(*iii*) Find $\log_7\left(\frac{1}{49}\right)$

(*iv*) Find possible values that satisfies the equation $\log_a 25 = 2$

(3) (*i*) Find ${}^9C_9$

(*ii*) How many words can make using the letters of the word BADULLA.

(*iii*) For any given values for α and β, state weather following identity is true or false.

$$sin(\alpha + \beta) = sin\alpha + sin\beta$$

(4) (*i*) Solve the equation $3x^2 - 0.5x + 2.1 = 0$

(*ii*) If $Z_1 = 1 - i$ and $Z_2 = 1 + i$, then find Z_1Z_2 & $\frac{Z_1}{Z_2}$

(*iii*) For any complex numbers Z, prove that $Z\bar{Z} = |Z|^2$

(5) (*i*) Find $sin\left(\frac{11\pi}{2}\right)$

(*ii*) Prove the identity $\frac{1+tan\propto}{1+cot\propto} = \frac{sin\propto}{cos\propto}$

(*iii*) In the triangular standard notation prove that,

$$a(cosC - cosB) = 2(b - c)cos^2\left(\frac{A}{2}\right)$$

(6) (*i*) If $t = tan\left(\frac{\theta}{2}\right)$, then prove $\cos\theta = \frac{1-t^2}{1+t^2}$

(*ii*) a, b, c & d are positive number such that $a > b$ & $c > d$

Then show $2(ab + 1) > (a + 1)(b + 1)$ if $a > 1$ & $b > 1$

(*iii*) Solve $|3x - 4| = x + 5$ for x

(*iv*) Show $\log_a b = \frac{1}{\log_b a}$ where $a, b > 0$ & $a, b \neq 1$

(7) (*i*) Prove ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

(*ii*) Expand $\left(\frac{1}{2} - x\right)^{-2}$ up to four terms.

(*iii*) From among a group of six men and nine women, how many three- member committees contain only men or only women.

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