

**Instructions to candidates**

Duration: 01 hour

Mark allocation: 100

Answer all the questions



01. Let  $\phi = 4xyz^3 - 3x^2y^2z$  and  $\varphi = x^2yz^2 + xy^2z$ .

- a) Determine  $grad \phi$  at the point (1,3,2). (marks 10)
- b) Find the directional derivative of  $\phi$  at the point (1,3,2) in the direction of the vector  $\vec{i} + 2\vec{j} - 3\vec{k}$ . (marks 15)
- c) Evaluate  $\nabla(\phi\varphi)$  at the point (1,3,2). (marks 15)  
[Hint : You can use the property  $\nabla(\phi\varphi) = \phi(\nabla\varphi) + \varphi(\nabla\phi)$ ]

02. a) Evaluate the line integral  $\int_C \vec{F} \cdot d\vec{r}$ , where  $\vec{F}(x, y, z) = x\vec{i} - z\vec{j} + y\vec{k}$  and C is given by

$$\vec{r}(t) = 2t\vec{i} + 3t\vec{j} - t^2\vec{k}, \quad -1 \leq t \leq 1. \quad \text{(marks 20)}$$

b) Show that  $div(curl \vec{u}) = 0$ , for the vector  $\vec{u} = x^2y\vec{i} - xyz\vec{j} + yz^2\vec{k}$ . (marks 20)

c) Use the Stoke's Theorem to evaluate  $\iint_S curl \vec{F} \cdot ds$ , where  $\vec{F}(x, y, z) = xz\vec{i} + yz\vec{j} + xy\vec{k}$  and

S is the part of the sphere  $x^2 + y^2 + z^2 = 4$  that lies inside the cylinder  $x^2 + y^2 = 1$  above the xy – plane. (marks 20)