Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
300 Level 1st Semester Examination –Jul./Aug. 2016
SCT 301-2 Advanced Mathematics I

Uva Wellassa University

Instructions to candidates
Duration: Two(02) hours
Number of questions: Four(04) Essay Questions
Mark allocation: 150 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions

1.

a. Use the definition of Laplace transform to prove $L\left\{e^{at}\right\}=\dfrac{1}{s-a}$. (10 mark)

b. Determine the Laplace transforms of the following functions.

i. $3e^{2t}-5e^{-4t}$ (03 mark)
ii. $\sin 4t+\cos 5t$ (03 mark)
iii. t^3+2t^2-6t+7 (03 mark)
iv. $4e^{2t}+3\sinh 4t$ (03 mark)

c. State the **first shift theorem** and hence find $L\left\{e^{-3t}\sin 7t\right\}$. (05 mark)

d. Determine the Laplace transform of $\dfrac{\sin at}{t}$. (06 mark)

e. Find the inverse transform of following functions.

i. $\dfrac{5}{s^2+25}$ (02 mark)
ii. $\dfrac{12s}{s^2+4}$ (02 mark)
iii. $\dfrac{3s+4}{s^2+9}$ (03 mark)

f. Express $F(s)=\dfrac{5s^2-4s-7}{(s-3)(s^2+4)}$ as partial fractions and hence determine its inverse transform.

(10 mark)

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2.

a. Sketch the graph of the following function and find its Laplace transform. (08 mark)

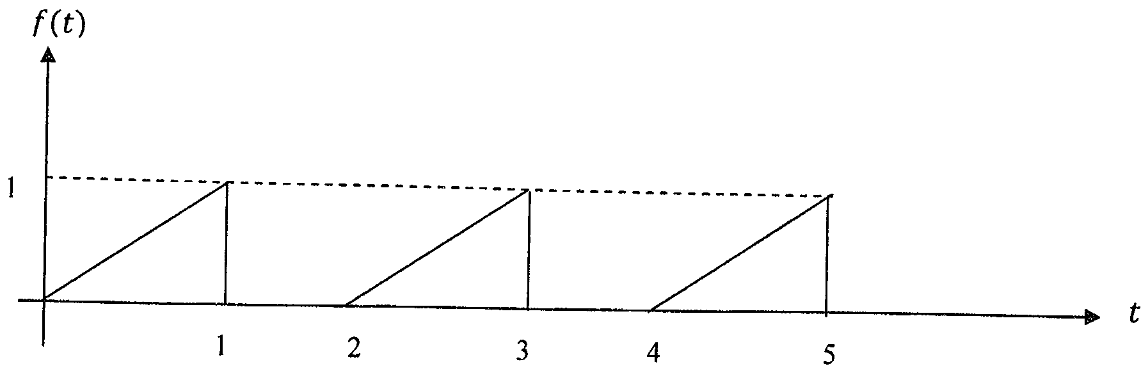
$$f(t)=\begin{cases}4t & \text{if } 0<t<2\\ 8 & \text{if } \quad t>2\end{cases}$$

b. Determine the function $f(t)$ whose transform $L(f(t))=F(s)$ is

$$F(s)=\frac{1}{s}\left\{2-5e^{-s}+8e^{-3s}\right\}.$$

Then sketch the graph of function $f(t)$ between $t=0$ and $t=4$. (12 mark)

c. Find the Laplace transform of the following periodic function. (12 mark)



d. Determine $f(t)=L^{-1}\left\{\frac{3(1-e^{-s})}{s(1-e^{-3s})}\right\}$ and hence sketch the resulting waveform of $f(t)$.

(08 mark)

3.

a. Solve the following ordinary differential equations by using Laplace transform.

i. $\frac{dx}{dt}+3x=e^{-2t}$ at $t=0,\ x=2$ (08 mark)

ii. $3\dot{x}-4x=\sin 2t$ at $t=0, x=\frac{1}{3}$ (08 mark)

iii. $\ddot{x}-6\dot{x}+8x=e^{3t}$ at $t=0, x=0, \dot{x}=2$ (12 mark)

b. Solve the following pair of simultaneous equations by using Laplace transform, where x and y are functions of t and given that at $t=0,\ x=0$ and $y=0$. (12 mark)

$$\dot{y} - x = e^{t}$$
$$\dot{x} + y = e^{-t}$$

4. Determine the Fourier series represention for the following periodic function defined by

$$f(x) = \begin{cases} -2 \text{ if } -\pi < t < 0 \\ 2 \text{ if } 0 < t < \pi \end{cases}$$

$$f(x+2\pi) = f(x).$$

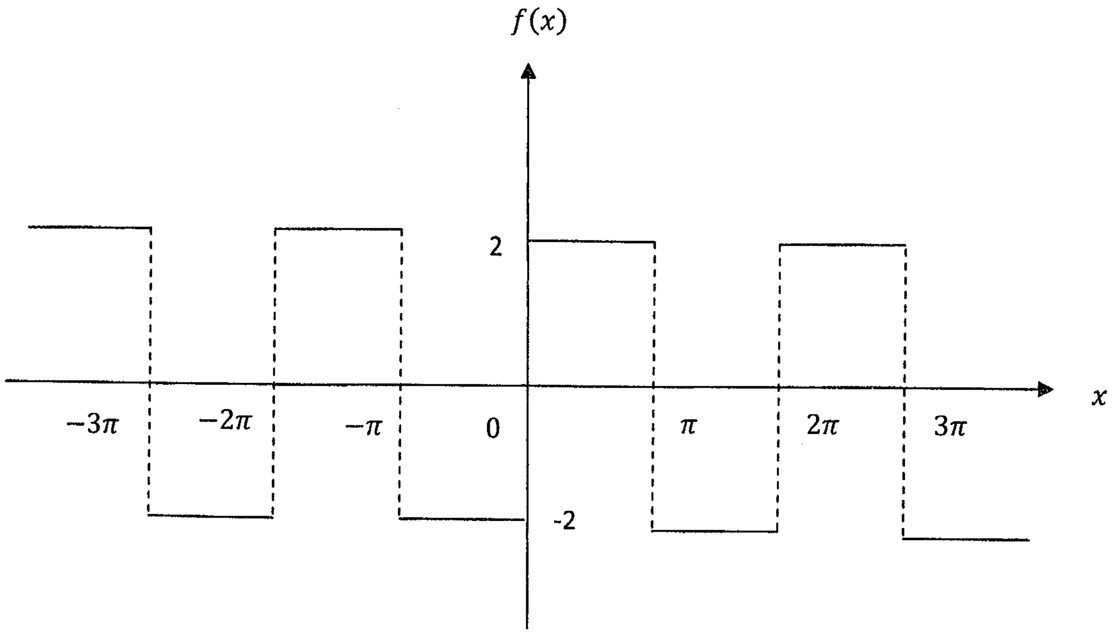


Figure 4.1: The graph of $f(x)$

(20 mark)

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