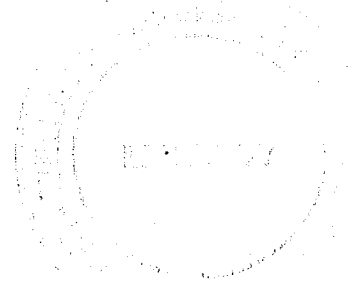




Uva Wellassa University, Sri Lanka
Btech Science and Technology
End Semester Examination- Semester 1
January -2009
MAT 201-2 – Calculus (Repeat)



Answer for Four (04) questions only

Time: Two hours

No. of questions: Five (05)

Calculators are acceptable

All expressions are given in standard mathematical notations

You may use standard notations

Any new notation or abbreviations you use must be clearly defined

1. a. A rectangle has an area equal to 25 cm². Its width is equal to x . Find the perimeter of the rectangle as a function of x .

(05 marks)

- b. An approximate value in billions for the world population, $p(t)$, t years after 1990 is given by

$$p(t) = \frac{10}{1 + e^{-0.03t}}$$

- i. What was the population in 1990?

(04 marks)

- ii. What will be the population in 2100?

(06 marks)

- c. Find the domain of the function f defined by $f(x) = \frac{5x}{\sqrt{2x-10}}$ and represent it as an inequality.

(10 marks)

2. a. Write down the negations of following mathematical statements.

- i. $p = q$ ii. $y < 2$

(06 marks)

- b. Given that, **A: Set of real numbers**
 B: Set of even numbers

Then, determine the truth value of following statements.

- i. $B \subset A$
ii. $A = B$
iii. $B \in A$

(03 marks each)

- c. Construct a truth table and prove following logical equivalence.

$$(p \Rightarrow q) \vee (q \Rightarrow p) \equiv \sim p \vee q \vee \sim q \vee p \equiv T$$

(10 marks)

3. a. Determine the following limits,

i. $\lim_{x \rightarrow 0} \frac{1}{\cos x}$

iii. $\lim_{x \rightarrow \infty} \frac{2x^2 - x + 1}{4x^2 - 3x - 1}$

ii. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

iv. $\lim_{t \rightarrow \infty} \frac{50}{2 + 3e^t}$

(03 marks each)

b. At what points, the function $f(x)$ is discontinuous, If

$$f(x) = \frac{1}{x-3} + 3x$$

(05 marks)

c. $f(x) = \begin{cases} x^3 - 4x^2 + 3x - 1 & ; x \neq 2 \\ c & ; x = 2 \end{cases}$

Find the number c that makes $f(x)$ continuous for every x .

(08 marks)

4. a. Differentiate the following functions with respect to x .

i. $y = x^2 \sin x$

ii. $y = \frac{\sin x}{1 + \cos x}$

iii. $y = e^{2x+1}$

iv. $y = \ln(x-4)$

(03 marks each)

b. The gas law for a fixed molar amount n of an ideal gas at absolute temperature T , pressure P and volume V is $PV = nRT$ where R is the gas constant. Show that

$$\frac{\partial P}{\partial V} \frac{\partial V}{\partial T} \frac{\partial T}{\partial P} = -1$$

(08 marks)

c. If $f(x) = 3x^3 - 4x^2 + 6x - 1$, Find $f'(x)$, $f''(x)$

(05 marks)

5. a. Evaluate the following integrals.

i. $\int \left(5e^{-2x} + \frac{9}{x} \right) dx$

ii. $\int \frac{1}{\sqrt{\sin^2 x + \cos^2 x}} dx$

iii. $\int \frac{2x}{\sqrt{x^2 + 4}} dx$

iv. $\int (10x^6 + 4x^4 - 3) dx$

(04 marks each)

b. i. $\int_0^{\pi/4} \sin x$

ii. $\int_1^2 3x^2 \cos(x^3 + 2) dx$

(4.5 marks each)