Uva Wellassa University, Sri Lanka.
End Semester Examination - August 2010
SCT 105-2 Mathematics I
(Repeat paper)

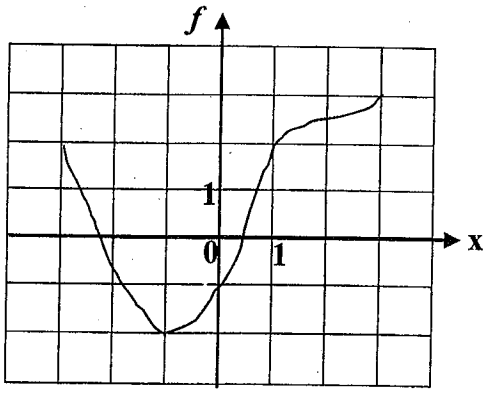
Time : Two (2) hours

Answer all questions.
Calculators are allowed.
Total four (4) pages.

1. What is the major difference between independent variable and dependent variable.

(2 marks)

a) The graph of a function f is given.



i. State the value of $f(-1)$.

ii. For which values of x , $f(x)$ is 2.

iii. State the domain and the range of f.

iv. On which interval is f increasing.

v. On which interval is f decreasing

(10 marks)

[P.T.O]

b) A spherical balloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$.

Find a function that represents the amount of air required to inflate the balloon from a radius from r inches to a radius of $r+1$ inches.

(5 marks)

c) The gravitational force exerted by the Earth on a unit mass at a distance r from the center of the planet is given by following equation.

$$F(r) = \begin{cases} \frac{GMr}{R^3} & if \quad r < R \\ \frac{GM}{r^2} & if \quad r \geq R \end{cases}$$

Where M is the mass of the Earth, R is its radius and G is the gravitational constant.

Is F a continuous function of r?

(5 marks)

[P.T.O]

2. What is the difference between series and sequences.

(2 marks)

a) If Rs. 1000 is invested at 6% compounded annual interest, then after n years the total amount is a_n .

i. Derive an expression for a_n.

ii. Find the first three terms of the sequence $\{a_n\}$.

iii. Is the sequence is convergent or divergent? Explain.

(12 marks)

b) A certain ball has the property that each time it falls from a height h on to a hard level surface, it rebounds to a height rh, where $0 < r < 1$. Suppose that the ball is dropped from an initial height of H meters. (Use the fact that the ball falls $\frac{1}{2}gt^2$ meters in t seconds with g gravitational force)

i. Assuming that the ball continues to bounce indefinitely, find the total distance it travels.

ii. Calculate the total time that the ball travels.

iii. Suppose that each time the ball strikes the surface with velocity (v) it rebounds with the velocity $(-kv)$, where $0 < k < 1$. How long will it take for the ball to come to rest.

(12 marks)

c) The size of an undisturbed fish population has been modeled by the formula;

$$p_{n+1} = \frac{b\,p_n}{a+p_n}$$

Where $p_n$is the fish population after n years and a and b are positive constants that depend on the species and its environment. Suppose that the population in year 0 is $p_0 > 0$.

i. Show that if $\{p_n\}$ is convergent, then the only possible values for its limits are 0 and $b - a$.

ii. Show that $p_{n+1} < (b/a)\,p_n$.

iii. Use previous part (ii) to show that if $a > b$, then $\lim_{n\to\infty} p_n = 0$, and explain it.

(12 marks)

[P.T.O]

3. Point out the major purpose of having differentiation (2 marks)

a) Suppose that the population for a certain bacteria $B(t)$ is given by,
$B(t) = 2000 + 3t + 2t^2 + 1.5t^3$; *where t is the time by seconds*

i. Find and interpret $B(0)$.

ii. Find $\frac{dB}{dt}$, *when* $t = 20s$, and interpret it.

(8 marks)

b) In a fish farm, a population of fish is introduced into a pond and harvested regularly. A model for the rate of change of the fish population is given by,

$$\frac{dP}{dt} = r_0\left(1 - \frac{P(t)}{P_c}\right)P(t) - \beta P(t)$$

Where r_0 is the birth rate of the fish, P_c is the maximum population that the pond can sustain, β is the percentage of the population that is harvested and $P(t)$ is the fish population.

i. What is the value of $\frac{dP}{dt}$ corresponds to a stable population.

ii. If the pond can sustain 10,000 fish, the birth rate is 15%, and the harvesting rate is 4%, find the stable population level.

iii. What happens, if β is raised to 5% .

(14 marks)

c) A large fish tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then the volume V of water remaining of the tank after t minutes as,

$$V(t) = 5000\left(1 - \frac{t}{40}\right)^2 \quad ; \quad 0 \le t \le 40$$

i. Find the rate at which the water drains from the tank at any time t.

ii. Find the draining rate at 5th minute.

iii. At what time the flowing is fastest. (*Hint:– Graph* $V'(t)$ *with respect to* t)

iv. At what time the flowing is slowest .

(16 marks)