Uva Wellassa University, Sri Lanka
End Semester Examination – August 2011
EAG 102-0
Mathematics for Biological Sciences

Time: Two (02) hours

Uva Wellassa University

Total **04** Questions
Answer **all** questions

1. (a) Define the terms **domain** and **range** of a function.

[04 marks]

(b) Find the domain and the range of the following functions by sketching the graph:

i. $y = \sqrt{4 - x^2}$ ii. $y = x^2 + 1$

[12 marks]

(c) Given $f(x) = \frac{x}{x+1}$; $x \neq -1$ and $g(x) = x^2 + 1$ find each of the followings:

i. $f\left(\frac{1}{2}\right)$ iii. $f(g(x))$ v. $g\left(f(g(0))\right)$

ii. $g(-1)$ iv. $f(f(x))$ vi. $g\left(f(f(0))\right)$

[12 marks]

2. (a) Find each of the following limits, if they exist:

i. $\lim_{x\to 3} \frac{x^2-2x-3}{x-3}$ iii. $\lim_{u\to\infty} \frac{5-3u}{6u+1}$ v. $\lim_{v\to 27} \frac{v-27}{\sqrt[3]{v}-3}$

ii. $\lim_{t\to 2} \frac{t^2-4}{t+2}$ iv. $\lim_{x\to-\infty} \frac{3x^2-2x-1}{x^3+4}$ vi. $\lim_{x\to 0} \frac{x^4+5x-3}{2-\sqrt{x^2+4}}$

[12 marks]

(b) Determine whether the following functions are continuous at the given point c:

i. $f(x) = \frac{3}{x-2}$ $at\ c = 2$

ii. $g(t) = \begin{cases} \frac{t-1}{\sqrt{t}-1}, & if\ t > 1 \\ 5 - 3t, if\ -2 \leq t \leq 1 \\ \frac{6}{t-4}, & if\ t < -2 \end{cases}$ $at\ c = -2$

iii. $h(x) = \begin{cases} 3x - 5, if\ x \neq 1 \\ 2, \quad if\ x = 1 \end{cases}$ $at\ c = 1$

iv. $p(x) = \begin{cases} \frac{x^2-x}{x^2-1}, if\ x \neq 1 \\ 1, \quad if\ x = 1 \end{cases}$ $at\ c = 1$

[12 marks]

[P.T.O.

3. (a) Differentiate each of the following functions with respect to x.

i. $f(x) = -\frac{1}{4}x^8 + \frac{1}{2}x^4 - 3^2$

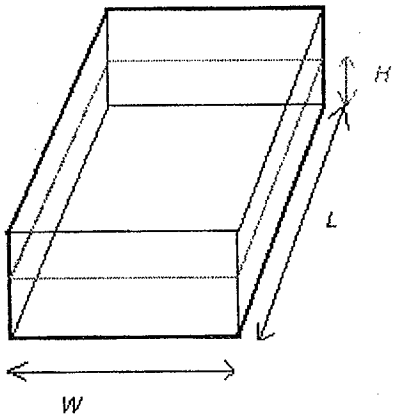
ii. $g(x) = \sqrt{4x^2 - x}$

iii. $h(x) = (x^3 - 6x)(2\tan x - 4x^3$

iv. $q(x) = \frac{4\sin x}{x^3+3}; x \neq \sqrt[3]{-3}$

[08 marks

(b) A rectangular water tank (see figure below) is being filled at the constant rate of 20 liters / second. The base of the tank has dimensions w = 1 meter and L = 2 meters. What is the rate of change of the height of water in the tank?



[08 mar

(c) A rectangular block has a square base. Its total surface area is 150.

i. If the base length is x , show that the volume V of the block is

$$V = \frac{75x - x^3}{2}$$

ii. Find the dimensions of the block when the volume is maximum.

[08 ma

4. (a) Compute each of the following integrals:

i. $\int x(4x^2 - 3x^{\frac{1}{2}} + 3)\, dx$

ii. $\int \frac{4u^5 - u^3 + u^2 - 3}{3u^3} du \; ; u \neq 0.$

iii. $\int \frac{e^{-x}+2}{e^{-x}} dx$

iv. $\int_0^2 (6x^2 - 4x + 5)\, dx$

v. $\int_0^{\pi/2} (\sin\theta - \cos\theta)\, d\theta$

[12 ma

(b) A proton moves in an electric field such that its acceleration (in cm^{-2}) is $a = -20(1+2t)^{-2}$, where t is in seconds.
Find the velocity as a function of time if $v = 30cms^{-1}$ when $t = 0$.
[Hint: The velocity of an object at time t, given the acceleration a, is given by $v = \int a\,dt$]

[06 marks]

(c) The average value of the function $y = f(x)$ from $x = a$ to $x = b$ is given by $y_{ave} = \frac{\int_a^b f(x)dx}{b-a}$.
The temperature T (in °C) recorded during a day followed the curve $T = 0.001t^4 - 0.280t^2 + 25$, where t is the number of hours from noon $(-12 \leq t \leq 12)$. What was the average temperature during the day?

[06 marks]