

**Instructions to candidates**

Duration: 02 hours

Number of questions: 04

Number of questions to be answered: 04

Mark allocation: 100

All symbols carry their usual meaning.



- 1.
- a. What do you mean by the following?
- A position vector,
  - A displacement vector,
  - Average and instantaneous velocity vectors and
  - Average and instantaneous acceleration vectors, of a particle moving on a  $xy$  plane.
- (10 mark)
- b. A rabbit run across a parking lot on which a set of coordinate axes has been generated using
- $$x = -0.21t^2 + 6.2t + 28$$
- $$y = 0.25t^2 - 8.1t + 20,$$
- At  $t = 2.5$  s, what is the rabbit's position vector  $\vec{r}(t)$ ?
  - Find the magnitude of vector  $\vec{r}$
  - What is the angle it makes with the  $x$  axis?
  - Graph the rabbit's path from  $t = 0$  to  $t = 25$ s.
  - Find the velocity vector  $\vec{v}(t)$  at time  $t = 15$ s.
- (10 mark)
- c. i. Write equations for the centripetal force  $F$  and the period of revolution  $T$  experienced by an object of mass  $m$  undergoing a circular motion in a circle with radius  $r$  and velocity  $v$ .
- (5 mark)

2.

- a. By considering a particle moving on  $xyz$  coordinate system with mass  $m$  and velocity  $v$ , define the angular momentum  $\vec{l}$  of this particle about the origin  $O$ .

(5marks)

- b. Hence, prove the angular form of the Newton's second law for this particle.

(5 marks)

- c. A Penguin of mass  $m$  falls from rest at point  $A$ , which is situated distance  $D$  away from the origin  $O$  of  $xyz$  coordinate system (the positive direction of the  $z$  axis is directly outward from the  $xy$  plane).

- i. What is the angular momentum  $\vec{l}$  of the falling penguin about the origin  $O$  after time  $t$ .

(5 marks)

- ii. About the origin  $O$ , what is the torque  $\vec{\tau}$  on this penguin due to the gravitational force  $\vec{F}$ .

(5 marks)

- iii. Hence, argue to show that if  $D$  and  $m$  are increased,  $\vec{\tau}$  is also increased linearly.

(5 marks)

3.

- a. What is "precession" in relation to a simple gyroscope.

(5 marks)

- b. Why does a spinning gyroscope stays aloft (in the air) upward instead of falling over like the non-spinning gyroscope and undergoes a precession?

(10 marks)

- c. Obtain an expression for a precession rate  $\Omega$  of a rapidly spinning gyroscope.

(5 marks)

- d. A certain gyroscope consists of a uniform disk with a 50 cm radius mounted at the centre of an axis that is 11 cm long and of negligible mass. The axis is horizontal and supported at one end. If the disk is spinning around the axis at 1000 rev/min, what is the precession rate?

(5 marks)

4.

- a. What is a simple harmonic motion (SHM)?

(5 mark)

- b. Obtain expressions for velocity  $v(t)$ , acceleration  $a(t)$ , and the total mechanical energy  $E$  of an object experiencing a SHM.

(10 marks)

- c. A block whose mass  $m$  is 680 g is fastened to a spring where spring constant  $k$  is 65 N/m. The block is pulled a distance 11 cm from its equilibrium position at  $x = 0$  on a frictionless surface and released from the rest at  $t = 0$ . Find the angular frequency, the frequency, the amplitude, the maximum speed and the maximum acceleration of the resulting motion.

(10 mark)

