Uva Wellassa University

Faculty of Management

Degree of Bachelor of Business Management in Entrepreneurship and Management

THIRD YEAR SECOND SEMESTER EXAMINATION –AUGUST/SEPTEMBER 2011

EMG 336-3 Actuarial Statistics-II

Instructions to candidates:

No. of pages : Eleven (11)
No. of questions : Twenty (20) MCQs
: Five (05) Structured Essay
: Three (03) Essay
Time allocation : Three (03) Hours
Marks allocation : 100 Marks

Index Number:

This is an open book examination

Students are provided with computer facilities in a lab with software R and Minitab

Part C – Essay Questions

Answer **only Two (02)** questions from Part C.

Marks allocation: 50 Marks

For each selected question, carry out the analysis/calculations using a computer. Copy the necessary parts of the output and paste in to a Microsoft Word document and label them appropriately. Name your document by your index number and save on the desktop so that the supervisor can copy it at the end of the examination. Write the answers (hand written) referring to the outputs wherever necessary.

01. An engineer is interested in estimating the probability that a particular electrical component will last at least 12 hours before failing. In order to do this, a random sample of 20 components was tested to destruction and their failure times $x_1, x_2 \ldots x_{20}$ were recorded. The engineer models failure times by assuming that they come from a distribution with probability density function f, given below.

$$f(x) = \frac{\alpha - 1}{(1+x)^{\alpha}} \quad , \alpha > 1, x > 0$$

The observed failure times (hours) are given below.

46.4, 1.9, 4.6, 48.7, 2.8, 0.5, 3.2, 0.1, 1.0, 490.1, 1.6, 12.6, 11.9, 1.1, 20.1, 1.2, 189.5, 35.1, 6.0, 1.4

i) Determine $\hat{\alpha}$, the maximum likelihood estimator of α, analytically or graphically. (Hint: $\hat{\alpha}$ is in between 1 and 5).

ii) Let X be the lifetime of a randomly selected component. Then, $P(X \leq x) = 1 - \frac{1}{(1+x)^{\alpha-1}}$.

Using your estimate, calculate the probability that a particular electrical component will last at least 12 hours before failing.

(25 Marks)

02. An actuary is interested in estimating the probability that the time between two accidents in a certain industrial plant is more than 30 days. He has obtained the times (days) between 41 previous consecutive accidents. Note: this gives 40 values.

35, 19, 63, 54, 22, 15, 81, 59, 65, 51, 14, 77, 8, 19, 31, 37, 30, 11, 47, 38, 38, 5, 44, 19, 32, 78, 26, 23, 52, 11, 16, 119, 64, 57, 51, 32, 85, 48, 100, 4

Let X be the time (days) in between two consecutive accidents.

i) Construct an approximate 95% confidence interval for the mean of X.

ii) Fit a suitable distribution for X. Hence calculate P(X > 30).

(25 Marks)

03.

i) From a pilot survey of 10 adults, it was found that 4 supported the policies of a particular political party. What sample size would be needed in order to be 95% confident that the true level of support is within 0.1 of the estimated proportion, assuming that the pilot survey provides us with a reasonable estimate of the true support?

ii) A random variable has a normal distribution with unknown mean μ and unknown variance σ^2. A $(1-\alpha)100\%$ confidence interval for the variance σ^2, using a sample of size n, is given by

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,(n-1)}}, \frac{(n-1)S^2}{\chi^2_{(1-\alpha/2),(n-1)}}$$

A pilot survey of 10 observations gave $S^2=25$. Determine the sample size n required to get a 95% confidence interval of width 20. (Hint: the answer is in between 10 and 100.)

(25 Marks)