Uwa Wellassa University, Sri Lanka.

End Semester Examination - June 2009

) Basic Calculus

MAT 101-0

Time : Two (2) hours

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Answer all questions.

Total three (3) pages.

Calculators are allowed.

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1. Point out the difference between an algebraic expression and an algebraic equation.

(2 marks)

a) An individual wants to use milk and orange juice to increase the amount of calcium and vitamin-A in her daily diet. An ounce of milk contains 30 mg of calcium and 50 mg of vitamin-A. An ounce of orange juice contains 5 mg of calcium and 60 mg of vitamin-A. How many ounce of milk and orange juice should he drinks each day to provide exactly 325 mg of calcium and 800 mg of vitamin A.

(7 marks)

b) Two pumps are used to fill a milk packet. One pump can fills the packet by itself in 9 seconds and the other can fills it in 6 seconds. How long it will takes if both pumps are operating together.

(7 marks)

c) How many liters of a mixture contains 80% of alcohol should be added to 5 liters of a 20% alcohol solution to yield a 30% solution.

(8 marks)

2. Define the word "function" in algebra.

(2 marks)

a) When a bird hovers in the air by flapping its wings, it uses energy. The power it uses is a function of its length. If we let $x$ as the length of the bird and $P$ the power used, the formula, which defining $P$ as a function of $x$ has the form $P = Cx^{\left(\frac{7}{2}\right)}$. Here C is a positive constant. If the bird's length doubles, by what ratio does the power it uses increase.

(7 marks)

b) Suppose that the length (in millimeters) of a small animal after $t$ days from the birth is given by;

$$h(t) = \frac{100}{2+3e^{-t}}$$

i. What is the length of the animal at birth.
ii. What is the eventual length ( the length when $t \to \infty$) of the animal.

(6 marks)

c) The size of an undisturbed fish population has been modeled by the formula;

$$p_{n+1} = \frac{b\,p_n}{a+p_n}$$

Where $p_n$ is the fish population after $n$ years and $a$ and $b$ are positive constants that depend on the species and its environment. Suppose that the population in year 0 is $p_0 > 0$.

i. Show that if $\{p_n\}$ is convergent, then the only possible values for its limits are 0 and $(b-a)$.
ii. Show that $p_{n+1} < (b/a)\,p_n$ .
iii. Use previous part to show that, if $a > b$, then $\lim_{n\to\infty} p_n = 0$, and explain it.

(9 marks)

3. What is the major difference between an independent variable and a dependent variable.

(2 marks)

a) A manufacture knows that it costs Rs9,000 to produced 1000 toaster ovens a week and Rs12,000 to produce 1500 toaster ovens a week.

i. Express the cost as a function of the number of toaster ovens produced, assuming that it is linear. Then sketch the graph.
ii. What is the slope of the graph and what does it represent.
iii. What is the y-intercept of the graph and what does it represent.

(8 marks)

b) In a certain country, income tax is assessed as follows. There is no tax on income up to Rs10,000. Any income over Rs10,000 is taxed at a rate of 10% up to on income Rs20,000. Any income over Rs20,000 is taxed at 15%.

i. Sketch the graph of the tax rate $(R)$ as a function of the income $(I)$.
ii. How much tax is assessed on an income of Rs26,000.
iii. Sketch the graph of the total assessed tax $(T)$ as a function of income $(I)$.

(8 marks)

c) A parking lot charges Rs50 for the first hour (or part of an hour) and Rs20 for each succeeding hour (or part) up to a daily maximum of Rs200.

i. Sketch the graph of the cost of parking at this lot as a function of the time parked there.

ii. Discuss the discontinuities of this function.

(7 marks)

4. Point out the major purpose of having differentiation.

(2 marks)

a) Suppose that the population for a certain bacteria $B(t)$ is given by,

$B(t) = 2000 + 3t + 2t^2 + 1.5t^3$ ; *where t is the time by seconds*

i. Find and interpret $B(0)$.

ii. Find $\frac{dB}{dt}$, *when* $t = 20s$, and interpret it.

(7 marks)

b) A graphic artist is designing a poster, which is to have margins of 2 inches at top, left and right sides, and 3 inches at the bottom. In order to save expenses, artist wants the total area of the poster to be as small as possible, but the printed area (the part inside the margins) has to be 180 square inches. What dimensions will minimize the total area.

(8 marks)

c) Gas is being pumped into a spherical balloon at a steady rate $3cm^3$ per second.

i. Find the volume of the balloon after 8 seconds.

ii. Find the radius of the balloon after 10 seconds.

iii. How fast is the radius increasing when the radius is $8cm$.

(10 marks)