

Instructions to candidates

Duration: Two(02) hours

Number of questions: Four(04) Essay Questions

Mark allocation: 100 mark

Use standard symbols without definition.

Scientific calculators are allowed.

Answer all questions

1.

a. Use L' Hospital's rule to evaluate the following limits.

i. $\lim_{x \rightarrow 1} \frac{x^3 + 3x - 4}{2x^2 + x - 3}$ (05 mark)

ii. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$ (05 mark)

b. Find $\lim_{x \rightarrow 2} f(x)$, where:

$$f(x) = \begin{cases} 2x+1, & x < 2 \\ x+3, & x \geq 2 \end{cases} \quad (04 \text{ mark})$$

c. Let $f(x) = \begin{cases} -x; & x < -1 \\ 1; & -1 \leq x \leq 1 \\ x; & x > 1 \end{cases}$, then show that f is continuous at $x = 1$ and $x = -1$.

(06 mark)

2.

a. Obtain the first five terms of each of the following sequences.

i. $\{n^2\}_{n=1}^{\infty}$ (04 mark)

ii. $\left\{ \frac{2n+3}{3n+4} \right\}_{n=1}^{\infty}$ (04 mark)

iii. $\left\{ \frac{n^2 + 3n + 5}{2n^2 + 5n + 7} \right\}_{n=1}^{\infty}$ (04 mark)



b. The first few terms of a sequence (x_n) are given below. Assuming that "natural pattern" indicated by these terms persists. Give the formula for the n^{th} term of x_n .

i. 5, 7, 9, 11, ..., (02 mark)

ii. 1, 4, 9, 16, ..., (02 mark)

iii. $\frac{1}{2}, \frac{-1}{4}, \frac{1}{8}, \frac{-1}{16}, \dots$, (02 mark)

c. From the $\varepsilon - N$ definition, show that $\lim_{x \rightarrow \infty} \frac{1}{n} = 0$. (05 mark)

d. Determine whether each of the following sequences are **convergent** or **divergent**.

i. $\left\{ \frac{n^2 + 3n + 5}{2n^2 + 5n + 7} \right\}_{n=1}^{\infty}$ (04 mark)

ii. $\{n + 1\}_{n=1}^{\infty}$ (03 mark)

3.

a. Determine whether each of the following series are **convergent** or **divergent**.

i. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ (05 mark)

ii. $1 + 2 + 4 + 8 + \dots$ (05 mark)

b. If $f(x, y) = 8x^3 + 7y^2 - 3xy$, then find f_x, f_y, f_{xx}, f_{yy} and f_{xy} . (10 mark)

4.

a. Determine which of the followings are propositions. If a given statement is a *proposition*, then write down its *truth value*.

i. Red is a color. (02 mark)

ii. $x + 1 = 5$ (03 mark)

iii. $5 \neq 4$ (03 mark)

iv. Read this question carefully. (02 mark)

b. Construct a truth table for the following statements.

i. $q \vee (p \vee \sim q)$ (10 mark)

ii. $(p \wedge q) \vee p \equiv p$ (10 mark)