

Uva Wellassa University, Sri Lanka
 Faculty of Science and Technology
 Science and Technology Degree program
 1st Semester Examination – March/April 2013



SCT 101-2 / SCT 104-2 Essential Mathematics

Instructions to candidates

Answer all **Four (04)** questions
 Time allocation: **Two (02)** hours
 Total marks allocated: 100



1.

i. Find general solutions of quadratic equation

$$ax^2 + bx + c = 0 ; \text{ where } a \neq 0$$

ii. Solve $2^x - 2^{-x} = 1$ (Hint: Take $t=2^x$)

iii. Find the partial fraction decomposition of $\frac{x^2+7x-2}{x^3-x}$

(25 marks)

2.

i. If $z=1-i$ and $w=1+i$ then calculate $zw(z+i)$

ii. Simplify the expression $\left(\frac{e^x+e^{-x}}{2}\right)^2 + \left(\frac{e^x-e^{-x}}{2}\right)^2$

iii. For any $a > 1$ prove that $a^x = e^{x \ln a}$

iv. For any $a, b > 1$ prove that $\log_a b = \frac{1}{\log_b a}$

(25 marks)

3.

i. Solve $x^3 < x^2 + 6x$

ii. For any $n \geq 3$ show that ${}^n C_3 = {}^n C_{n-3}$

iii. From among a group of six men and nine women, how many three-member committees contain only men or only women.

(25 marks)

4.

i. Use binomial formula to expand $(p-2q)^4$

ii. Show that

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \dots$$

iii. Prove that

$$\sin\theta\cos^3\theta + \sin^3\theta\cos\theta = \frac{1}{2}\sin 2\theta$$

(25 marks)