Uva Wellassa University, Sri Lanka
B.Tech. Degree Programme - 2007/08
BSc. (Computer Science) Degree Programme – 2007/08
End Semester Examination- Semester II
June/July - 2008

MAT 102-0 Basic Calculus

Answer **Four (4)** questions only
Time: **Two (2)** Hours
Formula sheet is provided at the last page

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(1) (i) Function h is defined by $h = \frac{3x^2-7x}{\sqrt{x}}$. Find followings

(a) $h(1)$

(b) $h(x+1)$

(c) $h(y)$

(ii) Let $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-2}$

(a) Find $f \circ g$ and $g \circ f$

(b) Find implied domains of $f \circ g$ and $g \circ f$

(iii) When a bird hovers in the air by flapping its wings, it uses energy. The power it expends (uses) is a function of its length. If we let x denote the length of the bird and P the power expanded, the formula defining P as a function of x has the form $P = Cx^{(\frac{7}{2})}$. Here C is a positive constant. If the bird's length doubles, by what ratio does the power it expends increase?

(2) (i) Find discontinuous points of the function $f(x) = \frac{x+1}{x^2-4}$

(ii) Find following limits

(a) $\lim_{x\to 0} \frac{\sin x}{\tan x}$

(b) $\lim_{x\to 1} \frac{x^3-1}{x-1}$ (Hint: take $a^3 - b^3 = (a-b)(a^2+ab+b^2)$)

(c) $\lim_{x\to 0} \frac{1-\cos 2x}{12x^2}$

(iii) Suppose that the length (in millimeters) of a small animal after t days from birth is given by $h(t) = \frac{100}{2+3e^{-t}}$

(a) What is the length of the animal at birth?

(b) What is the eventual length (the length when $t \to \infty$) of the animal?

(3) (i) Find the derivative of given functions with respect to x

(a) $y = x^4 - 3x^3 + 2x - 1$

(b) $y = x^2 \sin x$

(c) $y = \sin(x^2)$

(ii) Find implicit differentiation of following with respect to x

$$x - 2y^2 = 3\cos 2x$$

(iii) If $f(x,y) = \sqrt{x+y} + x\tan y$ then find $\frac{\partial^2 f}{\partial x \partial y}$

(4) (i) If the cost of manufacturing x items is $C(x) = x^3 + 20x^2 + 90x + 15$, find the marginal function and find the marginal cost at $x = 50$.

(ii) A graphic artist designing a poster, which is to have margins of 2 inches at the top, left and right sides, and 3 inches at the bottom. In order to save expenses, artist wants the total area of the poster to be as small as possible, but the printed area (the part inside the margins) has to be 180 square inches. What dimensions will minimize the total area?

(iii) Gas is being pimped into a spherical balloon at a steady rate of 3 cm^3 per second.

(a) Find the volume of balloon after 11.3 seconds?

(b) Find the radius of balloon after 11.3 seconds?

(c) How fast is the radius increasing when the radius 11.3 cm ?

(5) (i) Integrate following functions

(a) $\int \left(3e^x + \frac{2}{1+x^2}\right) dx$

(b) $\int \frac{15x^2+20x+5}{x^3+2x^2+x} dx$

(c) $\int_1^2 x^3\sqrt{x^4+5}\, dx$ (Hint: take $u = x^4 + 5$)

(ii) Find the area under the curve $f(x) = \sin x$ on the interval $[0, \pi]$. Where x measured in radians.

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(iii) A funny Joke begins to circulate by e-mail. Assume that the "infection" rate at which it spreads measured by the number of new people receiving the Joke per day is given by

$$J'(t) = e^t - 0.9 + 15t$$

If the Joke was originally sent to 36 people, find $J(t)$, which the number of people who have received the Joke after t days.

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