Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
200 level 2nd Semester Examination – Dec./Jan. 2018/19
SCT 202 – 3 Differencial Equations and Applications

Uva Wellassa University

Instructions to candidates
No. of pages : Two (02)
No. of questions : Four (04) Essay questions
Time allocation : Two (02) hours
Marks allocation : 100 mark
Use Standard symbols without definition
Scietific Calculators are allowed
Answer All Questions

Part A – Theory

01.

a. Show that if c_1 and c_2 are constants, then $y = (c_1 + c_2x)e^{-x} + e^x$ is a solution of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4e^x$. (3 mark)

b. Solve the following differential equations.

i. $\frac{dy}{dx} = \frac{2-e^{-x}}{3+2y}$, $y(0) = 0$ (4 mark)

ii. $x^2\frac{dy}{dx} = 3xy + 2y^2$ (4 mark)

iii. $\frac{dy}{dx} + \frac{4}{x}y = x^3y^2$, $y(2) = -1$ (4 mark)

c. A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and the water entering the tank has a salt concentration of $\frac{1}{5}(1 + \cos(t))$ lbs/gal. If a well mixed solution leaves the tank at a rate of 6 gal/hr, how much salt is in the tank when it overflows? (10 mark)

(36)

02.

a. Solve the following homogeneous equations.

i. $y'' - 8y' + 17y = 0 \; ; \; y(0) = -4 \, , \; y'(0) = -1.$ (5 mark)

ii. $y'' - 4y' + 4y = 0 \; ; \; y(0) = 12 \, , \; y'(0) = -3.$ (6 mark)

b. Find the general solution of the following non homogeneous equations.

i. $y'' - 4y' = xe^x + \cos(2x)$ (7 mark)

ii. $y'' + 9y = \tan(3t)$ (7 mark)

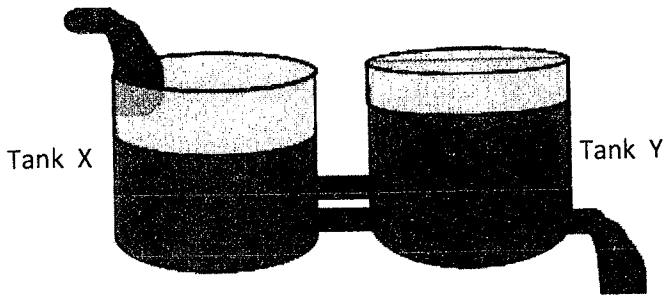
03.

a. Consider the second order equation $y'' - 10y' + 25y = 0 \; ; \; y(0) = 2 \text{ and } y'(0) = -5 \, .$

i. Rewrite it into a system of first order equations. (6 mark)

ii. Solve the system you obtained in part (i). (12 mark)

b. Someone has connected two tanks together and labeled as tank X and tank Y, as shown in the following Figure.



Tank X initially have 100 gallons of brine made with 100 pounds of salt. Tank Y initially has 100 gallons of pure water. Pure water is pumped into tank X at a rate of 2.0 gallons per minute. Some of the mixture of brine and pure water flows into tank Y at 3 gallons per minute. To keep the tank levels the same, one gallon of the Y mixture flows back into tank X at a rate of one gallon per minute and 2.0 gallons per minute drains out.

If x(t) be the amount of salt in tank X and y(t) the amount of salt in tank Y, we obtain the following system:

$$\frac{dx}{dt} = \frac{y}{100} - \frac{3x}{100}$$
$$\frac{dy}{dt} = \frac{3x}{100} - \frac{3y}{100}$$

Find the amount of salt at any given time in both tanks. (12 mark)

04.

a. Show that, the functions $u(x) = 1$ and $v(x) = cos\left(\frac{n\pi x}{L}\right)$ are orthogonal over $[-L, L]$.

(8 mark)

b. Find the Fourier Series of the function below.

$$f(x) = \begin{cases} 1 + x \ , -2 < x < 0 \\ 1 - x \ , \ 0 < x < 2 \end{cases} \quad \text{with period} = 4$$

(12 mark)