Algorithmic Aspects of Trees in Graph Theory: Identification and Applications

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Introduction

With the rapid advances of the computer technology, Mathematics and Computer Science have developed into two entirely separate disciplines. Since graphical approach lend themselves naturally as model for a variety of situations, including the hardware of computer and certain aspects of computer software, Graph Theory plays an increasingly important role within Computer Science. In the study of graphs, it soon becomes evident that efficient algorithms are necessary for solving problems of any significant magnitude. It has become fashionable to mention that there are applications of Graph Theory to some areas of Physics, Chemistry, Computer technology, Electrical and civil engineering, Sociology and Economics.

Computers have made our daily activities easier with fast and highly accurate outcome. Since we find the applications of Graph Theory in almost all areas, using the computer to solve practical problems makes our task easy. Transforming of our problems into a form that can be understood by the computer is the task of a computer language. Algorithms are used for that purpose. This research consists of Algorithmic aspect of Trees in Graph Theory and is designed to emphasize the close tie between the theoretical and algorithmic aspect of Trees in Graph Theory. Rather than employing a specific computer language for presenting algorithms, the steps were used that are common to many languages. This research has introduced few algorithms, based on properties of Trees in Graph Theory, for practical problems which we often come across. A complexity analysis is also given for each algorithm.

The main objective is to recognize the algorithms to solve real world problems using the properties of Trees in Graph Theory and the other objective is to apply these algorithms to solve some problems in day to day life.

Methodology

Mainly research can be divided into four sections and two phases.

3.1 Phase 1: Since it is of paramount importance to solve real world problems, the problems that exist should be identified. Then these problems were transformed as Graph Theoretic (Tree) Problems.

3.2 Phase 2: Since the objective was to use algorithms to solve these problems, the algorithms that exist should be recognized. Then those algorithms can be applied to solve the identified real world problems.

Results and discussion

Problem Statement 1
The weighted graph G of Figure 1(a) shows six cities and the cost of building roads between certain pair of cities. Road system that will connect the six cities at the lowest cost needed to be built.

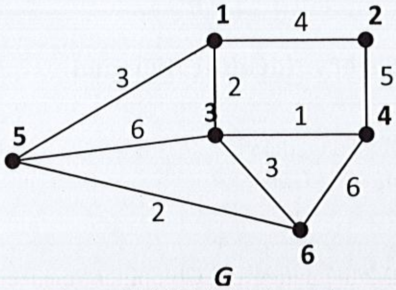

Figure 1(a) Weighted graph

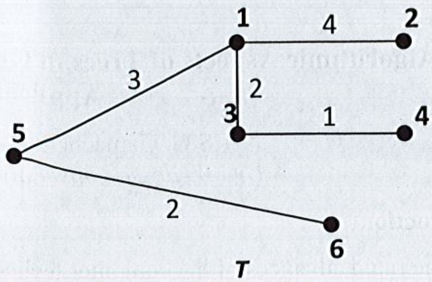

Figure 1(b) A spanning tree

The solution must be a minimal spanning tree since it must contain all vertices and the sum of whose weight is minimum. The tree T shown in Figure 1(b) is a minimal spanning tree which has a weight of 12.

Kruskal's Algorithm

To determine a minimum spanning tree in a non trivial connected weighted (p, q) graph G,

Step 1: $S \leftarrow \Phi$

Step 2: let e be an edge of minimum weight such that $e \notin S$ and $S \cup \{e\}$ is acyclic, and let $S \leftarrow S \cup \{e\}$.

Step 3: If $|S| = p-1$, then output S ; otherwise, return to Step 2.

Problem Statement 2

The most common way to represent characters internally in a computer is by using fixed-length bitstrings (0's and 1's. For example, ASCII (American Standard Code for Information Interchange) represents each character by a string of seven bits. Examples are given in Table 1.

Table 1: ASCII codes of some characters

Characters	ASCII Code
A	100 0001
1	011 0001
*	010 1010

Is it generally possible to represent strings of characters, such as text and programs, in less space than if ASCII was used?

Using Huffman Algorithm binary tree representing a prefix code whose codeword have minimum average weighted length was constructed.

Same algorithm to check what sequence of tests will minimize the expected time taken by the phone box to identify the coin can be used.

Problem Statement 3

Simulation of groundwater flow is a discipline where computer applications are extensively utilised. Numerous computer models have been developed to model fluid flow and solute transport by the collaboration of mathematicians and hydrogeologists. 3D sophisticated models such as MODFLOW or MT3D and 1D models of Barker Codes type are available worldwide (Anderson and Woessner, 1992; Udagedara, 2007). In order to make the model realistic, some unlikely assumptions are made. Isotropy and

homogeneity of the aquifer (medium) are the fundamental assumptions for any model. Thus, the model is always far behind the real aquifer conditions. Therefore, in order to rectify the errors associated with aforementioned fundamental assumptions, authors wish to put forward a solution with the application of graph theory. The research investigates the flow through a saturated medium where heterogeneity and anisotropy of the hydraulic conductivity of the medium have been taken in to the consideration.
The Dijkstra's Algorithm can be used to find the shortest path from vertex v1 to vertex vn in a weighted graph.
Other practical problems which can be solved by using Dijkstra's algorithm are:
1. Traffic information systems
2. OSPF- Open Shortest Path First, used in Internet routing.

References

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