Uva Wellassa University, Sri Lanka
B.Tech. Degree Programme – 2006/07
End Semester Examination- Semester 2
July -2008

CHE 283-1/PHY 261-1 QUANTUM MECHANICS

Answer four (4) questions only **Time: One (01) Hour**

1. Write equations showing the relationship among energy, *E*, wavelength, λ, velocity, *c*, and the wave number of an EM radiation. In an experiment, a beam of EM radiation in the microwave region of frequency 2.0×10^4 MHz is allowed to incident on a piece of soft matter. After some time, it was noticed that a wave pattern similar to that belongs to the incident wave has been formed (registered) on the surface of the soft matter. Given the length between the two troughs of the observed wave pattern on the surface of the soft matter is equal to 15 mm, calculate the speed, *c*, of the incident microwave radiation. Also calculate the energy, *E*, and the wave number of the incident microwave.

(25 marks)

2. What is the photoelectric effect? Give the Einstein's explanation for photoelectric effect using the idea of energy quantization. Given the work function, Φ, of Platinum (Pt) is 5.65 eV, calculate the maximum energy, *E*, of the photoelectrons emitted from a clean Pt surface when it is irradiated with soft x-ray radiation of wavelength 200 Å. Find the stopping potential and the threshold frequency for these electrons.

(25 marks)

3. Give the Born interpretation for the wave function of the time-dependent Schrödinger equation. Show that if $\Psi_1(x, t)$ and $\Psi_2(x, t)$ are solutions to the one-dimensional time-independent Schrödinger equation, then $\Psi_1 + \Psi_2$ is also a solution to the equation. A wave function has the form of $\Psi(x,t) = A(\sin x)e^{-2x}$ for $0 \le x \le 1$. Find A so that

$$\int_0^1 |\Psi(x,t)|^2 \, dx = 1$$

Hence find the normalized wave function. (25 marks)

4. By solving the 1-D time-independent Schrodinger equation for each case, show that although the energy of a particle confined into one-dimensional infinite potential well ("a particle in a box") is quantized, the energy of a particle moving in a free space with zero potential function is not quantized. Show clearly the boundary conditions you used to solve the 1-D Schrodinger equation in each case.

(25 marks)

5. The π electrons in conjugated hydrocarbons are modeled as a particle in one dimensional box. Butadiene is a conjugated hydrocarbon where there are 4 π electrons free to move along the molecule. Assuming that butadiene is a linear 1-D molecule,

(a) Calculate the length, *a*, of a butadiene molecule in angstroms if 9.02×10^{-19} J is required to excite a π electron from the ground state of butadiene to its first excited state. You can use any equations you derived elsewhere without deriving them here again.

(b) Excited π electrons deexcite to their ground states by releasing the absorbed energy. Given that there are no other intermediate processes occur, calculate the wavelength and the wave number of the radiation emitted in the deexcitation process of butadiene. Sketch the shape of the wave functions of the π electrons in the ground and the first excited states.

(25 marks)

6. Explain what do you mean by "quantum mechanical tunneling of electrons". Write an expression for the tunneling probability T (this is also known as transmission probability) when a stream of particle with energy E is impinging on a potential barrier with height V_0 where $E << V_0$.

Scanning tunneling microscope (STM) is an advanced instrument based on the concept of quantum mechanical tunneling of electrons. The tunneling current, *I*, generated in STM strongly depends on the distance between the sharp metal tip and the surface in question and varies according to the following equation.

$$I = e^{-KZ}$$

where *K* is a constant and the *Z* is the distance between the metal tip and the surface. Given K ~ 2.3 Å^{-1} for a particular case, show that decreasing the distance between the tip and the surface by 1 Å increases the tunneling current by one order of magnitude (10 times).

(25 marks)