

Instructions to candidates

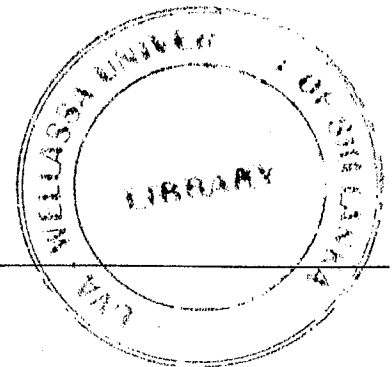
Duration: 02 hours

Number of questions: 4 essay questions

Mark allocation: 100 mark

Scientific calculators are allowed

Answer all questions



1.

a. Let $A = \begin{pmatrix} 2 & 5 & -2 \\ 3 & 4 & 3 \\ 4 & 2 & 1 \end{pmatrix}_{3 \times 3}$ $B = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & 4 \\ 3 & 1 & 5 \end{pmatrix}_{3 \times 3}$

i. Find product AB . (5 mark)

ii. Find determinant and transpose of matrix A. (10 mark)

b. Solve the following system of linear equations using **Gauss-Jordan elimination method**.

$$x - 2y + 3z = -1$$

$$y - 5z = 1$$

$$5x + 5y - 7z = 63$$
 (5 mark)

2.

a. Solve the following linear equations using **Integrating Factor method**.

i. $x \frac{dy}{dx} + (x + 1)y = 3$ (3 mark)

ii. $\frac{dy}{dx} = -\left(\frac{2}{x}\right)y + 10x^3$ (3 mark)

b. Solve the following exact differential equations.

i. $(8y - x^2y)dy + (x - xy^2)dx = 0$ (5 mark)

ii. $(2x^3 - 3x^2y + y^3)dy = (2x^3 - 6x^2y + 3xy^2)dx$ (5 mark)

c. Solve the following ordinary differential equations with constant coefficients.

i. $\frac{d^3y}{dx^3} + y = 0$ (5 mark)

ii. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ (4 mark)

3.

- a. Flair Furniture Company produces tables and chairs. Each product must go through a two stage manufacturing process; carpentry and painting. The data for the above problem is given below:

	Tables(per table)	Chairs(per chair)	
Profit contribution	\$7	\$5	Hours Available
Carpentry	3 hrs	4hrs	2400
Painting	2hrs	1hr	1000

- Make no more than 450 chairs
- Make at least 100 tables

Formulate a linear programming model to maximize the profit of the company.

(10 mark)

- b. Find the extreme points of the set of feasible solutions for the given linear programming problems and find the optimal solution.

$$\begin{aligned}
 \text{Min } z &= 2x_1 + 3x_2 \\
 \text{s. t } &3x_1 + x_2 \leq 6 \\
 &x_1 + x_2 \leq 4 \\
 &x_1 + 2x_2 \leq 6 \\
 &x_1, x_2 \geq 0
 \end{aligned}$$

(15 mark)

4. Let $\frac{dx}{dt} = \begin{pmatrix} -7 & 0 & 6 \\ 0 & 5 & 0 \\ 6 & 0 & 2 \end{pmatrix} \underline{x}$

- a. Compute the eigen values of $\frac{dx}{dt}$. (10 mark)
- b. Find the eigen vectors corresponding to each eigen values. (10 mark)
- c. Hence find the general solution for the above situation. (5 mark)