

**Uva Wellassa University of Sri Lanka**  
**Faculty of Science and Technology**  
**Department of Science and Technology**  
**400 level 1<sup>st</sup> Semester Examination – May / Jul. 2017**  
**SCT462-2 Control Theory**



**Instructions to candidates:**

**Duration:** Two (02) hours

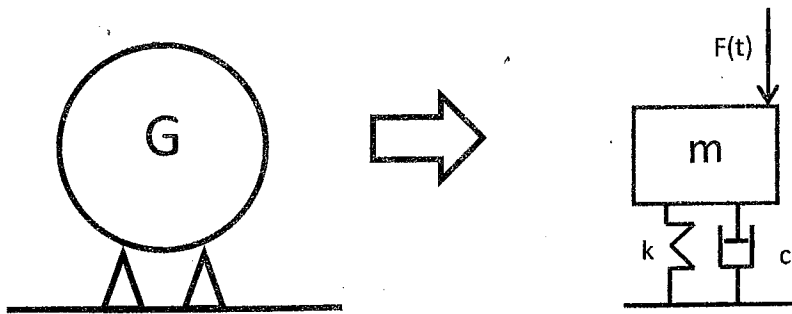
**Number of questions:** Four (04)

**Answer all questions**

**Some useful equations are shown in page 5**

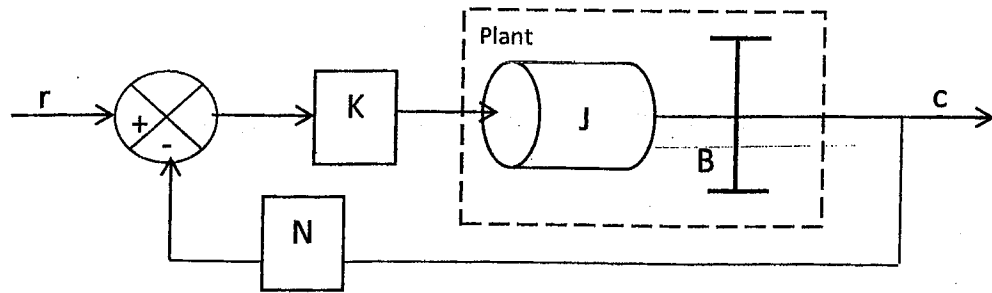
**Mark allocation:** 100

- 1) A Factory is supposed to install a generator (mass  $m$ ) in their building. The factory management wants to analyze the mechanical behavior of the machine before it is installed. Therefore, it is required to model the system. The generator can be simplifying as below. Where  $F(t)$  is excitation force,  $k$  is spring constant and  $c$  is damping ratio.



- a) Obtain the mathematical model of the system mentioning all the assumptions. Marks (10)
- b) Obtain the Laplace transformation of the system (take initial conditions as zero). Marks (15)
- 2) The servo system shown in the figure consists of a proportional controller and loaded elements (inertia and viscous-friction elements). Suppose that we wish to control the output position ( $c$ ) in accordance with the input position ( $r$ ).





- a) Get overall transfer function, using a block diagram. Consider the plant as

$$\frac{1}{s(Js+B)}$$

Marks (8)

b) Show that,  $G(s) = \frac{\omega_n^2 / N}{s^2 + 2\xi\omega_n s + \omega_n^2}$

Where,  $\omega_n^2 = \frac{KN}{J}$  and  $2\xi\omega_n = \frac{B}{J}$

Marks (17)

3)

- a) **Figure (a)** (in page 5) shows the unit step response of transfer function,

$$G(s) = \frac{25}{s^2 + 3s + 25}$$

Label the followings and clearly mark the numerical values on the plot.

The completed diagram should be attached to the answer script.

- i. Delay time
- ii. Rise time
- iii. Peak time
- iv. Maximum overshoot
- v. Setting time (both 2% and 5%)

Marks (5)

- b) Consider a system where  $\xi = 0.5$  and  $\omega_n = 6 \text{ rad/s}$ . Find the followings subjected to a unit step input.

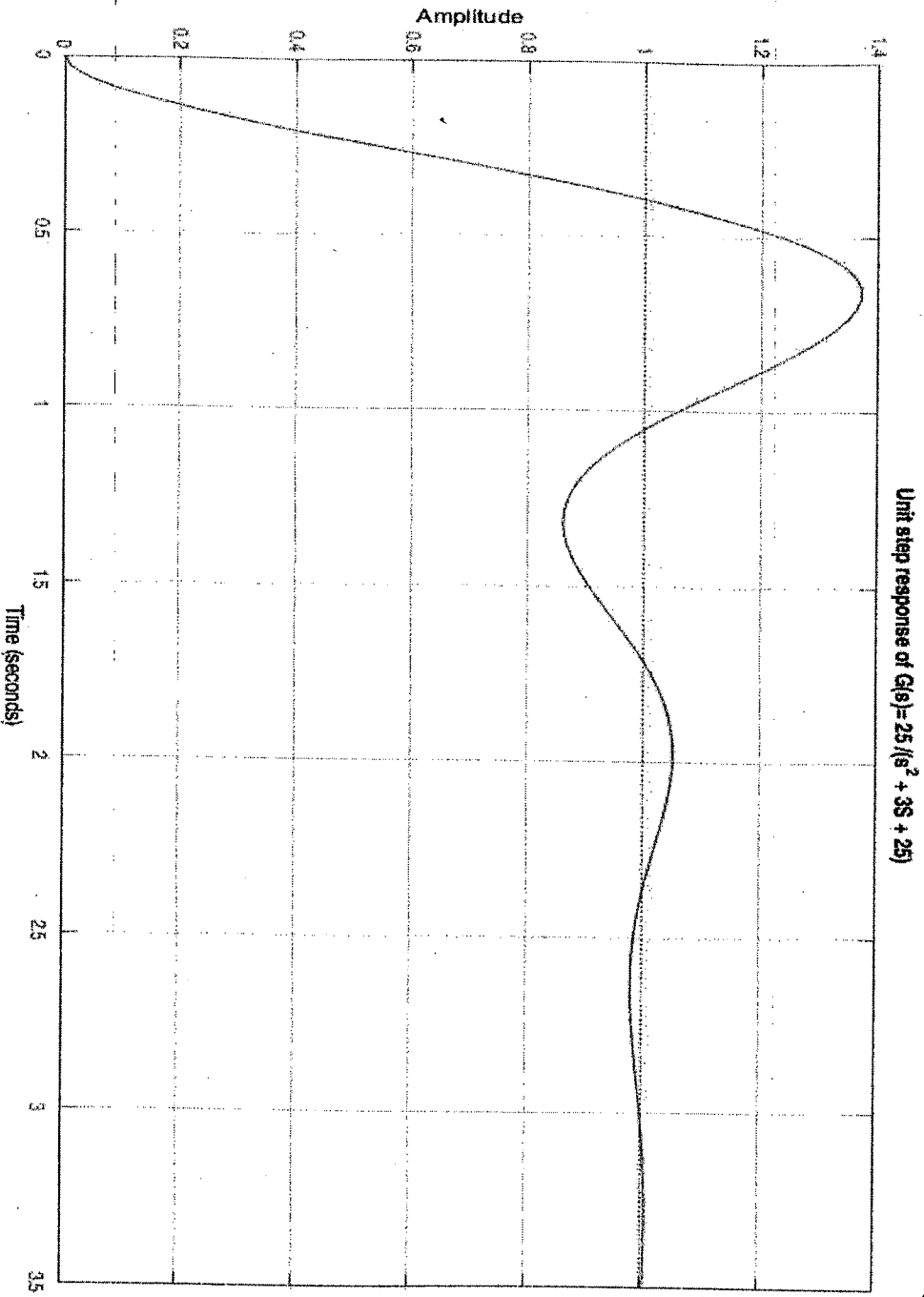
- i. Rise time
- ii. Peak time
- iii. Maximum overshoot
- iv. Setting time

Marks (8)

- c) Determine the values of  $\xi$  and  $\omega_n$  for following system such that respond to a step input with approximately 5% overshoot and with settling time 2s. (use the 2% criterion)

Marks (12)

**Figure (a) for Question no (3)**



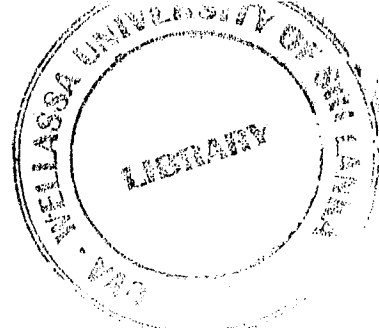
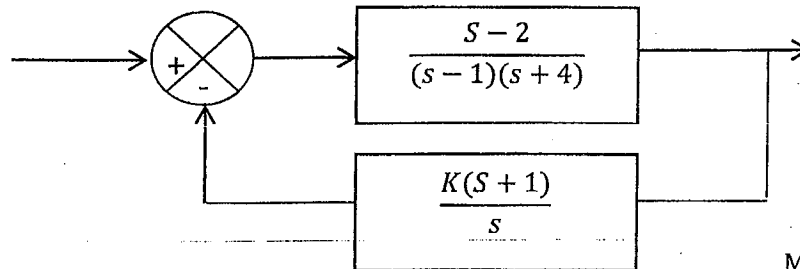


Table of Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. $e^{at}$	$\frac{1}{s-a}$
3. $t^n, n=1,2,3,\dots$	$\frac{n!}{s^{n+1}}$	4. $t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. $\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}}, n=1,2,3,\dots$	$\frac{1 \cdot 3 \cdot 5 \dots (2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t \sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10. $t \cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at) - at \cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12. $\sin(at) + at \cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at) - at \sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14. $\cos(at) + at \sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\frac{s \sin(b) + a \cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\frac{s \cos(b) - a \sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$	18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $e^{at} \sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20. $e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $e^{at} \sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22. $e^{at} \cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n e^{at}, n=1,2,3,\dots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
25. $u_c(t) = u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26. $\delta(t-c)$ Dirac Delta Function	$e^{-cs}$
27. $u_c(t) f(t-c)$	$e^{-cs} F(s)$	28. $u_c(t) g(t)$	$e^{-cs} \mathcal{L}\{g(t+c)\}$
29. $e^{ct} f(t)$	$F(s-c)$	30. $t^n f(t), n=1,2,3,\dots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t} f(t)$	$\int_s^\infty F(u) du$	32. $\int_0^t f(v) dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau) g(\tau) d\tau$	$F(s)G(s)$	34. $f(t+T) = f(t)$	$\frac{\int_0^T e^{-st} f(t) dt}{1-e^{-sT}}$
35. $f'(t)$	$sF(s) - f(0)$	36. $f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
37. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$		

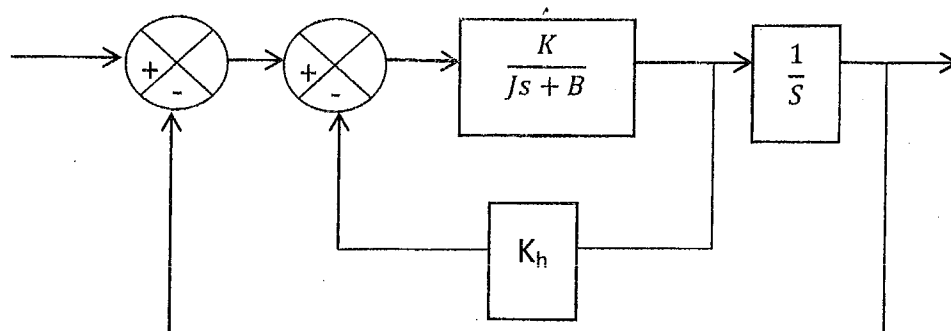
4)

- a) Determine the value of  $K$  for the following system when the system is stable.



Marks (8)

- b) For the system bellow determine  $K$  and  $K_h$  values so that maximum overshoot in the unit step input is 0.25s and peak time is 2s. Using the values of  $K$  and  $K_h$  obtain the rise time and settling time. Assume  $J = 1.5 \text{ kgm}^2$  and  $B = 1 \text{ Nm/rad s}$ .



Marks (9)

- c) Plot the route locus for  $G(s) = \frac{1}{s(s+4)}$  showing all the required steps.

Marks (8)

