(1)

Uva Wellassa University, Sri Lanka.

End Semester Examination - February \ March, 2011

SCT 301-2 Advanced Mathematics I

Time: Two (2) hours

Total three (3) questions.

Answer all questions.

Total four (4) pages.

Notes are allowed to use.

Calculators are allowed to use.

1. Fourier series can be used effectively in several fields such as, Electronic, Image Processing, Remote Sensing, etc. Write down 4 applications in the Fourier series.

(04 marks)

a) A Resistor-Capacitor (RC) circuit can be used as an Integrator; The Integrator is a type of a Low Pass Filter circuit (Figure 01) that converts a square wave input signal into a triangular waveform output. If the RC time constant (5τ) is long compared to the time period of the input RC waveform the resultant output will be triangular in shape. Moreover, the higher the input frequency the lower will be the output amplitude compared to that of the input (Figure 02).

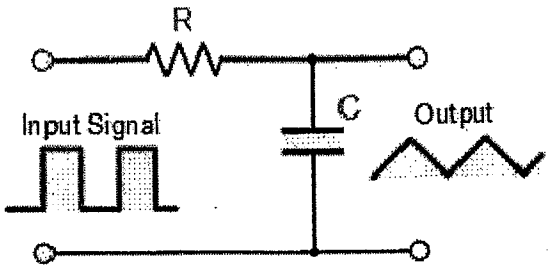

Figure 01: Low Pass Filter circuit

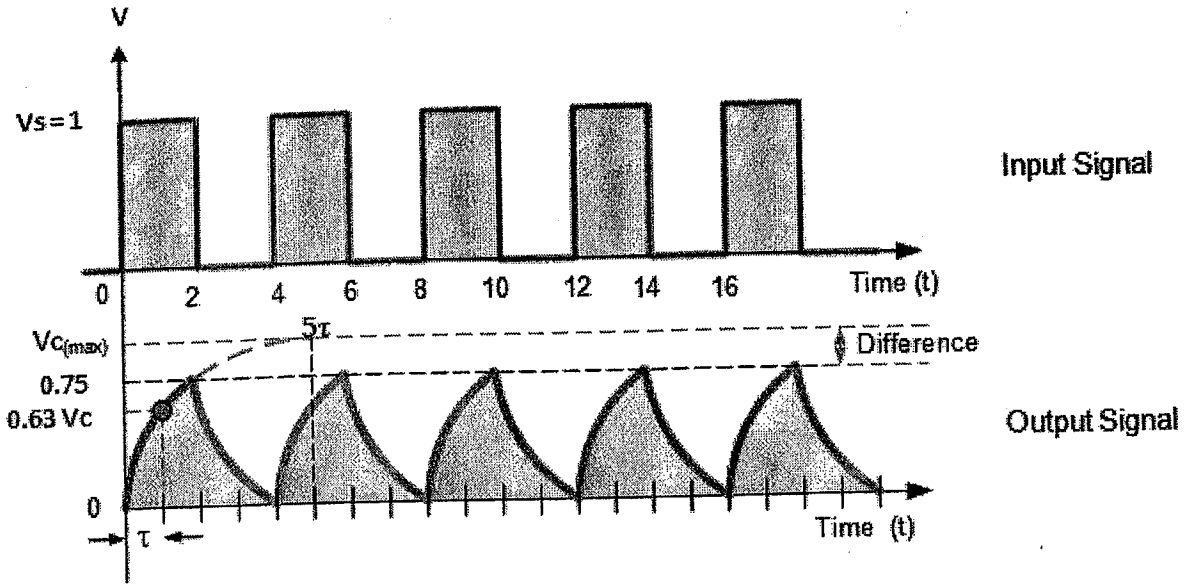

Figure 02: RC waveform input and output signals

As a researcher you are required to find Fourier series of the input and output signals, and compare them (Assume that output signal is triangular).

(30 marks)

2. Laplace Transformations is a very useful technique which is applicable in many fields of the Physics, Electronic and Engineering. Write down 4 applications in the Laplace Transform.

(04 marks)

a) A set of researchers, who are doing experiments in a new shock-absorber which is made by a spring with a mass of 2kg upon it. Moreover, it has natural length of 0.5m and a force of 25.6 N is required to maintain it stretched to a length of 0.7m. Suppose the spring is immersed in an oil with damping constant of c = 40. Find the position of the mass at any time t, if it starts from the equilibrium position and is given a push to start it with an initial velocity of 0.6 m/s.

(Hint: - Damping Force = $-c\frac{dx}{dt}$, Restoring Force = $-kx$)

(15 marks)

b) Find the charge and the current at time t in the bellow circuit (Figure 03) if $R = 40\Omega$, $L = 1$, $C = 16 \times 10^{-4}$, $E(t) = 100\cos 10t$, It is given that, the initial charge and current are zero.

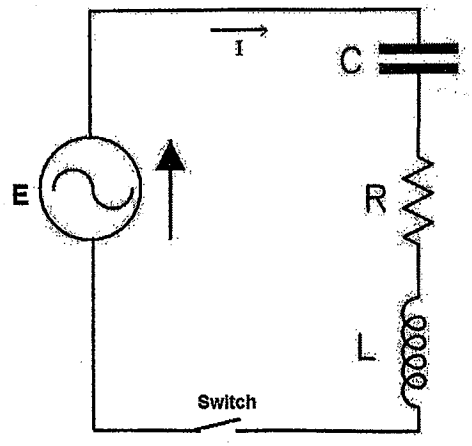

Figure 03: LCR Circuit

(Hint: - voltage drop across the capacitor = $\frac{Q}{C}$, $I = \frac{dQ}{dt}$; Where Q is the charge of the capacitor)

(15 marks)

3. Write down 2 applications for each wave equation and heat equation.

(04 marks)

a) Due to the earthquake hit to Japan, a Cable-Stayed bridge situates in Sendai got a huge oscillation. The length of the bridge is 100m, at t = 0 it had the shape $3\ sin\ \frac{\pi x}{50};\ 0 \leq x \leq 100$, Find the displacement of the bridge at any later time t.

(14 marks)

b) A steel rod which is having the length 1m is fixed in the engine of a train and two ends of that is maintained with zero temperature. While the train runs this rod get heated. Find the heat of the rod at any later time t, if when t = 0 the temperature of the rod is given as $20\ sin3\pi x - 2\ sin5\pi x;\ 0 \leq x \leq 1.$

(14 marks)