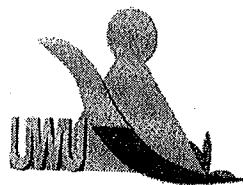


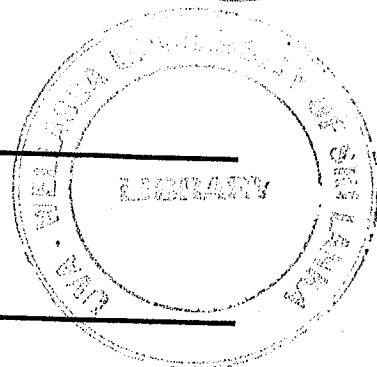
Uwa Wellassa University, Sri Lanka.
End Semester Examination - December 2009

SCT 201-2 Mathematics I

(Repeat paper)



Time : Two (2) hours



Answer all questions.

Calculators are allowed.

Total two (2) pages.

1. What is the purpose of having differentiation.

(02 marks)

a) In a study it was found that the temperature of sea water T at time t (measured in days) at a depth of x (measured in feet) can be modeled by the following function.

$$T(x, y) = T_0 + T_1 e^{-\lambda x} \sin(\omega t - \lambda x);$$

where $\omega = \frac{2\pi}{365}$ and λ , T_0 and T_1 are positive constants.

- Find the rate of change of temperature with respect to the depth.
- Find the rate of change of temperature with respect to the time.
- Show that $T(x, t)$ satisfies the heat equation $T_t = k T_{xx}$ for a certain constant k .

(15 marks)

b) The gas law for a fixed molar amount n of an ideal gas at absolute temperature T , pressure P and volume V is $PV = nRT$, where R is the gas constant.

Show that,
$$\frac{\partial P}{\partial V} \frac{\partial V}{\partial T} \frac{\partial T}{\partial P} = -1$$

(05 marks)

2. Explain what is meant by partial differentiation.

(02 marks)

a) The temperature at a point (x, y) on a flat metal plate is given by $(x, y) = \frac{60}{1+x^2+y^2}$, where T is measured in $^{\circ}\text{C}$ and x, y in meters. Find the rate of change of temperature with respect to distance at the point $(2, 1)$, in following directions.

- The x direction.
- The y direction.

(12 marks)

- b) The wind-child index is modeled by the function

$$W = 13.12 + 0.6215 T - 11.37 v^{0.16} + 0.3965 T v^{0.06}$$

Where T is the temperature ($^{\circ}\text{C}$) and v is the wind speed (km/h).

- i. Find rate of changing W with respect to T .
- ii. Find rate of changing W with respect to v ,
when $T = 20^{\circ}\text{C}$ and $v = 30\text{km/h}$.

(12 marks)

3. What is the major difference between independent variable and dependent variable.

(02 marks)

- a) A manufacture finds that it costs \$9,000 to produced 1000 toaster ovens a week and \$12,000 to produce 1500 toaster ovens a week.

- i. Express the cost as a function of the number of toaster ovens produced, assuming that it is linear. Then sketch the graph.
- ii. What is the slope of the graph and what does it represent.
- iii. What is the y-intercept of the graph and what does it represent.

(15 marks)

- b) In a certain country, incoming tax is assessed as follows. There is no tax on income up to \$10,000. Any income over \$ 10,000 is taxed at a rate of 10% up to on income &20,000. Any income over \$20,000 is taxed at 15%.

- i. Sketch the graph of the tax rate R as a function of the income I .
- ii. How much tax is assessed on an income of \$26,000.
- iii. Sketch the graph of the total assessed tax T as a function of income I .

(15 marks)

4. What is the difference between series and sequence.

(02 marks)

- a) The weight $\omega(x)$ (force due to gravity) of an object of mass m varies with it's altitude (length) x miles above the surface of the earth according to the function,

$$\omega(x) = \frac{mgR^2}{(R+x)^2} ; \text{ Where } R \text{ is the radius of the earth and } g \text{ is the acceleration due to gravity.}$$

- i. Express $\omega(x)$ as a infinite series.
- ii. When x is much smaller than R , show that $\omega(x) \cong mg \left(1 - \frac{2x}{R}\right)$.

(12 marks)

- b) Find the maclaurin series for $f(x) = e^x$.

(06 marks)