Uwa Wellassa University, Sri Lanka.
End Semester Examination - December 2009
SCT 201-2 Mathematics I
(Repeat paper)

Time : Two (2) hours

Answer all questions.
Calculators are allowed.
Total two (2) pages.

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1. What is the purpose of having differentiation.

(02 marks)

a) In a study it was found that the temperature of sea water T at time t (measured in days) at a depth of x (measured in feet) can be modeled by the following function.

$T\,(x\,,y) = T_o + T_1 e^{-\lambda x} \sin\,(\omega t - \lambda x)$;

where $\omega = \frac{2\pi}{365}$ and λ , T_o and T_1 are positive constants.

i. Find the rate of change of temperature with respect to the depth.
ii. Find the rate of change of temperature with respect to the time.
iii. Show that $T(x,t)$ satisfies the heat equation $T_t = k\,T_{xx}$ for a certain constant k.

(15 marks)

b) The gas law for a fixed molar amount n of an ideal gas at absolute temperature T, pressure P and volume V is $PV = nRT$, where R is the gas constant.

Show that, $\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1$

(05 marks)

2. Explain what is meant by partial differentiation.

(02 marks)

a) The temperature at a point (x,y) on a flat metal plate is given by $(x,y) = \frac{60}{1+x^2+y^2}$, where T is measured in ℃ and x, y in meters. Find the rate of change of temperature with respect to distance at the point $(2,1)$, in following directions.

i. The x direction.
ii. The y direction.

(12 marks)

b) The wind-child index is modeled by the function

$$W = 13.12 + 0.6215\,T - 11.37\,v^{0.16} + 0.3965\,T\,v^{0.06}$$

Where T is the temperature (℃) and v is the wind speed (km/h).

i. Find rate of changing W with respect to .

ii. Find rate of changing W with respect to v,

when $T = 20$℃ and $v = 30km/h$.

(12 marks)

3. What is the major difference between independent variable and dependent variable.

(02 marks)

a) A manufacture finds that it costs \$9,000 to produced 1000 toaster ovens a week and \$12,000 to produce 1500 toaster ovens a week.

i. Express the cost as a function of the number of toaster ovens produced, assuming that it is linear. Then sketch the graph.

ii. What is the slope of the graph and what does it represent.

iii. What is the y-intercept of the graph and what does it represent.

(15 marks)

b) In a certain country, incoming tax is assessed as follows. There is no tax on income up to \$10,000. Any income over \$ 10,000 is taxed at a rate of 10% up to on income &20,000. Any income over \$20,000 is taxed at 15%.

i. Sketch the graph of the tax rate R as a function o0f the income I.

ii. How much tax is assessed on an income of \$26,000.

iii. Sketch the graph of the total assessed tax Tas a function of income I.

(15 marks)

4. What is the difference between series and sequence.

(02 marks)

a) The weight $\omega(x)$ (force due to gravity) of an object of mass m varies with it's altitude (length) x miles above the surface of the earth according to the function,

$\omega(x) = \frac{mgR^2}{(R+x)^2}$; Where R is the radius of the earth and g is the acceleration due to gravity.

i. Express $\omega(x)$ as a infinite series.

ii. When x is much smaller than R, show that $\omega(x) \cong mg\left(1 - \frac{2x}{R}\right)$.

(12 marks)

b) Find the maclaurin series for $f(x) = e^x$.

(06 marks)