Uva Wellassa University, Sri Lanka
End Semester Examination – February 2011
SCT 351-3 Materials Physics

Time: Three (03) hours

Total **06** Questions
Answer **ALL** questions
Each question contains **50 marks**

Boltzmann constant - 1.38×10^{-23} JK^{-1}
Avogadro number - 6.022×10^{23} mol^{-1}
Plank's constant (h) - 6.626×10^{-34} Js
Electron charge (e) - 1.602×10^{-19} C
Electron mass (m) = 9.109×10^{-31} kg

PART - A

01) a. Obtain the wave function and the probability density function of a particle of mass m confined to move in a one dimensional box of length l. Show these in separate graphs. (25 marks)

b. Explain the comparison between classical and quantum ideas of the above three results. (10 marks)

c. What is the minimum energy possible for the particle? (7 marks)

d. Calculate the value of lowest energy of an electron in one dimensional force free region of length 4 $\dot{A}$. (8 marks)

02) (i) a. Explain the concept of wave particle dualism.

b. Wave properties of particles normally observe only when we study very small particles. Explain. (8 marks)

c. Show that the wavelength of an electron of rest mass m_0 moving with a velocity v is given by (8 marks)

$$\lambda = \frac{h}{mc} \frac{\left[1 - \frac{v^2}{c^2}\right]}{\frac{v}{c}}$$

hint : According to the theory of relativity, the mass m of a particle moving with velocity v is given by $m = \frac{m_0}{\left[1-\frac{v^2}{c^2}\right]^{1/2}}$ (9 marks)

(ii) a. What is normalization of a wave function? Normalize the one dimensional wave function given by

$$\Psi(x) = A \sin\left(\frac{\pi x}{a}\right) \qquad 0 < x < a$$
$$\Psi(x) = 0 \qquad \text{outside}$$

(8 marks)

b. What is a Hermitian operator ?

(8 marks)

c. Show that the eigenkets of any hermition operator are orthogonal to each other if the eigenvalues are different.

(9 marks)

03) a. Explain the free nature of the valence electrons in the free electron gas model.

(8 marks)

b. Define Fermi energy. Explain the behavior of Fermi-Dirac distribution function with change in temperature.

(12 marks)

c. Show that the Fermi level (E_{F_0}) for a three-dimensional free electron gas is given by,

$$E_{F_0} = \frac{\hbar^2}{2m}\left[\frac{3\pi^2 N}{V}\right]^{2/3}$$

$V =$ volume of the crystal
$N =$ Total number of free (valence) electrons

(20 marks)

d. Sodium (bcc) has a lattice parameter of 4.29 Å. Calculate the Fermi energy of sodium at absolute zero.

(10 marks)

04) Give short notes on the followings.

a. Photo electric emission
b. Quantum tunneling
c. Failures of the free electron gas model
d. Distinguish between Free electron theory and Band theory of solids
e. Distinguish between metals, insulators and semiconductors on the basis of the band theory

(10 × 5 marks)

PART - B

5) a. What is statistical ensemble?

(5 marks)

b. Write down three types of ensembles. Compare and contrast them.

(15 marks)

c. Explain the distinguishing features of Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics.

(10 marks)

d. What are the number of ways to arrange two particles x and y in three energy states according to Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics? Write in tabular form and explain.

(10 marks)

e. State which statistics (classical Maxwell-Boltzmann; Fermi-Dirac; or Bose-Einstein) would be obeyed by the followings and explain why.

i. Free electrons in metals
ii. Photons
iii. Molecules of N_2 gas at N.T.P.

(10 marks)

6) a. Maxwell-Boltzmann equation for distribution of energy among the molecules of an ideal gas is given by

$$n(E)dE = \frac{2\pi N}{(\pi kT)^{3/2}} E^{\frac{1}{2}} e^{-E/kT} dE$$

(where all the symbols have their usual meaning)

Starting from that obtain an expression for Maxwell-Boltzmann law of distribution of speeds.

(15 marks)

b. Obtain an equation for average speed ($\bar{v}$) of an ideal gas molecule.
(Use the standard integral $\int_0^\infty x^3 e^{-ax^2} dx = \frac{1}{2a^2}$)

(20 marks)

c. Calculate the value of average speed ($\bar{v}$) of a molecule of H_2 at 27 °C.

(15 marks)

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