Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
100 level 2nd Semester Examination – Dec. / Jan. 2018/19
SCT102-3 Calculus

Uva Wellassa University

Instructions to candidates
Duration: Three (03) hours
Number of questions: Six (06) Essay questions
Mark allocation: 200 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions.

1.

a. State whether the following statements are **True or False**. Justify your answer.

i. If a function f is not defined at $x = a$, then the limit $\lim_{x \to a} f(x)$ does not exist. (02 mark)

ii. If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then $\lim_{x \to a}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$. (02 mark)

iii. If $\lim_{x \to a^-} f(x) = l_1$ and $\lim_{x \to a^+} f(x) = l_2$, then $\lim_{x \to a} f(x)$ exists only if $l_1 = l_2$. (02 mark)

b. Evaluate the following limits,

i. $\lim_{z \to 4}\left(\dfrac{\sqrt{z} - 2}{z - 4}\right)$ (03 mark)

ii. $\lim_{h \to 0}\left(\dfrac{(6+h)^2 - 36}{h}\right)$ (04 mark)

iii. $\lim_{x \to -3}\left(\dfrac{\sqrt{2x+22} - 4}{x+3}\right)$ (04 mark)

c. Evaluate the following limits in terms of α, where $\alpha = \lim_{x \to 0}\left(\frac{\sin x}{x}\right)$.

i. $\lim_{x \to 0}\left(\frac{\sin 2x}{x}\right)$ (03 mark)

ii. $\lim_{x \to 0}\left(\frac{x+\sin 2x}{x+\sin 5x}\right)$ (03 mark)

d. Given that $7x \leq f(x) \leq 3x^2 + 2$, for all x. Use **Sandwich theorem** to determine the value of $\lim_{x \to 2} f(x)$.

(05 mark)

e. Let $f(x) = \begin{cases} 7-4x & ; \quad x \leq -6 \\ x^2+2 & ; \quad -6 < x \leq 1 \\ 3 & ; \quad x > 1 \end{cases}$

i. State whether $\lim_{x \to -6} f(x)$, and $\lim_{x \to 1} f(x)$ exist or not. Justify your answer.

(08 mark)

ii. Show that the function $f(x)$ is continuous at $x = 1$ and discontinuous at $x = -6$.

(04 mark)

2.

a. Find the slope of the tangent line to the curve $y = x^2 + 9$ at the point $P \equiv (1,1)$.
(04 mark)

b. Find the derivative of following functions with respect to x.

i. $y = \frac{6}{\sqrt{x^3}} + \frac{1}{8x^4} - \frac{1}{3x^{10}}$ (03 mark)

ii. $y = (x-4)(2x + x^2)$ (03 mark)

iii. $y = \frac{e^{2x}}{\sqrt{x}}$ (03 mark)

iv. $y = (\ln x . \sin x)^7$ (03 mark)

c. If $y = \log_{10} x$, find $\frac{dy}{dx}$. (04 mark)

d. Let $f(x) = x^3 - 3x^2$,

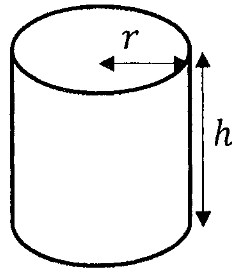
i. Find the intervals of x where $f(x)$ is increasing and decreasing. (04 mark)

ii. Find the value of local minimum or local maximum. (03 mark)

iii. Find the point of inflection. (02 mark)

iv. Sketch the graph of $f(x)$. (04 mark)

e. A cylinder must be to hold $(432\pi)cm^3$ of oil. The dimensions ($r = radius$ and $h = height$) are in centimetres.
Surface areas of the cylinder $= (A) = 2\pi rh + 2\pi r^2$
Volume of the cylinder $= (V) = \pi r^2 h$



i. Express the surface area of the cylinder in terms of r.
(02 mark)

ii. Hence find the dimensions $(r \& h)$ such that minimize the cost of the metal used to make the container.
(05 mark)

3.

a. Evaluate $\int \left(1+x^{3}\right)^{5} x^{2} dx$ by using a suitable **substitution.**

(05 mark)

b. Use **Integration by parts** to show, $\int \left(e^{x} \cos x\right) dx = \frac{1}{2}\left(e^{x} \cos x + e^{x} \sin x\right) + C$, where C is an arbitrary constant.

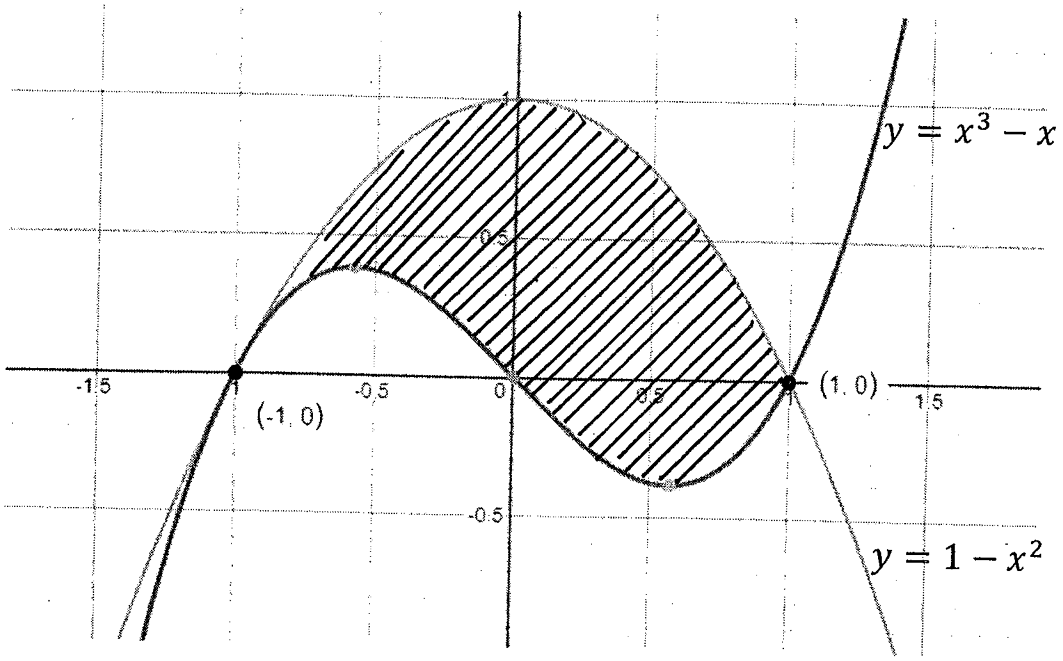
(05 mark)

c. Compute, $\int_{1}^{2} \left(\frac{2x}{3x^{2}+10x+3}\right) dx$.

(05 mark)

d. Find the shaded area that is bounded by the curves $y = 1 - x^2$ and $y = x^3 - x$ as indicated in following figure.

(10 mark)



4.

a. Obtain the first five terms of the following sequences.

i. $\left(\frac{2-(-1)^n}{n^3}\right), n \in \mathbb{N}$ (03 mark)

ii. $\left(\frac{(-1)^{n-1}}{3.6.9...(3n)}\right), n \in \mathbb{N}$ (03 mark)

iii. $\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^n}\right), n \in \mathbb{N}$ (03 mark)

b. Determine whether the following sequences are *convergent* or *divergent*. **Justify your answer**.

i. $(1), \ n \in \mathbb{N}$ (03 mark)

ii. $\left(\frac{1}{n}\right), n \in \mathbb{N}$ (03 mark)

iii. $\left(\frac{3n^2+2n+3}{n+1}\right), n \in \mathbb{N}$ (03 mark)

iv. $\left(10+(-1)^n\right), n \in \mathbb{N}$ (03 mark)

c. Determine whether the following series are *convergent* or *divergent*. **Justify your answer.**

i. $\sum_{n=1}^{\infty} 3^n$ (03 mark)

ii. $\sum_{n=1}^{\infty}\left(\frac{n^3+3n+6}{3n^3-2n+2}\right)$ (03 mark)

iii. $\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)(n+2)}\right)$ (03 mark)

5.

a. Use the **L'Hospital's rule** to find the following limits.

i. $\lim_{x \to 0}\left(\frac{e^x - x - 1}{\cos x - 1}\right)$ (04 mark)

ii. $\lim_{x \to -1}\left(\frac{\sqrt{x+10} + 3x^{\frac{1}{3}}}{4x^2 + 3x - 1}\right)$ (04 mark)

iii. $\lim_{x \to 0}\left(\frac{\sin x - x}{x^3}\right)$ (04 mark)

iv. $\lim_{x \to e}\left(\frac{1 - \ln x}{\frac{x}{e} - 1}\right)$ (04 mark)

b. State the **Rolle's Theorem** and verify the Rolle's Theorem for the function $f\colon [1,3] \to \mathbb{R}$, $f(x) = (x-1)^2(x-3)^5$ on the interval $[1,3]$.

(07 mark)

c. Find the **Taylor series** of the function $f(x) = e^x$ about $x = 0$.

(10 mark)

6.

a. Which of the following sentences are *propositions*. If a given statement is a proposition, then write down its *truth value*.

i. Answer this question. (02 mark)

ii. Colombo is the capital of Sri Lanka. (02 mark)

iii. What time is it? (02 mark)

iv. $x + 2 = 11$ (02 mark)

v. $3 + 2 = 6$ (02 mark)

b. What is the *negation* of each of the following propositions?

i.	3 is a natural number.	(02 mark)
ii.	$5 + 9 = 14$	(02 mark)
iii.	π is an irrational number	(02 mark)

c. Consider the following statements,

P: Black is a colour
Q: 4 is not a square number

Find the truth value of the following propositions based on above information.

i.	$\sim P$	(02 mark)
ii.	$P \wedge Q$	(02 mark)
iii.	$P \vee Q$	(02 mark)
iv.	$P \Rightarrow Q$	(02 mark)
v.	$P \Leftrightarrow Q$	(02 mark)

d. Construct the *truth table* for the statement, $(P \wedge [\sim(Q \vee R)]) \Rightarrow \sim P$.

(06 mark)