Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Science and Technology Degree Programme
1st Year 1st Semester Examination – August/September 2014
SCT 101-2 Essential Mathematics

Uva Wellassa University

Use standard symbols without definition.
Scientific calculators are allowed.
Number of Questions: Four (04)
Answer all questions
Time allocation: Two (02) hours
Mark allocation: 100 marks

1.

a. Derive the formula for the solutions to the quadratic equation $ax^2 + bx + c = 0\,;\, a \neq 0$

(5 mark)

b. Solve $\frac{x^2}{2} = 5x - 17$ (5 mark)

c. A radiator contains 25 liters of water and antifreeze solution. Where 20% (by volume) is antifreeze. How much of this solution should be drained and replaced with pure antifreeze for the new solution to be 60% (by volume) antifreeze?

(10 mark)

d. Find the partial fractions of $\frac{4x^3+10x+4}{x(2x+1)}$ (10 mark)

2.

a. If $z = 2 + i$ and $w = 3 - 5i$ then calculate $\frac{zw}{Z+w}$ and absolute value of it.

(8 mark)

b. For any $a, b > 1$ prove that $\log_a b = \frac{1}{\log_b a}$ (7 mark)

c. Solve the inequality

$$1 - x - 2x^2 < 0$$

(5 mark)

d. Solve $\log_3 x - \log_3 2 = 1 - \log_3(x+5)$ (5 mark)

3.

a. If $X = \begin{vmatrix} 2 & 1 & 0 \\ -1 & 0 & 3 \end{vmatrix}$ and $Y = \begin{vmatrix} 3 & -1 & 2 \\ 1 & 2 & 0 \\ -2 & 0 & 1 \end{vmatrix}$ find the product XY. (5 mark)

b. For any $n \geq 4$ show that ${}^nC_4 = {}^nC_{n-4}$ (5 mark)

c. There are 4 Mathematics books, 5 English books and 6 Science books. In how many ways can you arrange them so that books on the same subject are together? (5 mark)

d. A question paper consists of 10 questions divided into two parts A and B, containing 4 questions and 6 questions, respectively. A student is required to attempt 6 questions in all, selecting at least 2 from each part. In how many ways can a student select a question? (10 mark)

4.

a. Verify $(\sec x + \tan x)(1 - \sin x) = \cos x$ (5 mark)

b. Simplify

$$2\sin\left(A + \frac{\pi}{4}\right)\sin\left(A - \frac{\pi}{4}\right)$$ (5 mark)

c. Use binomial formula to expand $\left(x - \frac{x}{6}\right)^6$ (5 mark)

d. Find the constant term in the expansion of $\left(x^2 - \frac{2}{x}\right)^6$ (5 mark)