Uva Wellassa University, Sri Lanka

End of the Semester Examination- Second Semester 2007

MAT 101-2 Essential Mathematics

Answer for all Questions (4 Questions)
Time: Two (2) hours

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(1) (*i*) Find $\sin\left(\frac{8\pi}{3}\right)$ and $\cos 75°$.

(*ii*) Prove the identity $\frac{\cos ec\theta - \cot\theta}{\cos ec\theta + \cot\theta} = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$.

(*iii*) Find the x-range which satisfies the inequality $|3x+2| \leq 4-x$.

(*iv*) In the triangular standard notation prove that,

$$\sin\left(\frac{A-B}{2}\right) = \frac{a-b}{c}\cos\left(\frac{C}{2}\right)$$

(2) (*i*) Solve $(2x-1)^5 = -32$.

(*ii*) For any $x, y > 0$ prove that

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y \quad ; \quad \text{where } a > 0 \ \& \ a \neq 1.$$

(*iii*) Show that $\log_u\left(\frac{1}{\sqrt{u}}\right) = -\frac{1}{2}$; where $u > 0$ & $u \neq 1$.

(*iv*) Evaluate $2^{-3\log_2 5}$.

(3) (*i*) Use binomial formula to expand $(p-2q)^4$.

(*ii*) Show that $\sqrt{x+1} = 1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4+\ldots$

(*iii*) How many committees of 4 people are possible from a group of 9 people if;

a) There are no restrictions?

b) Both Nimal and Mary must be on the committee?

c) Either Nimal or Mary, but not both, must be on the committee?

(4) (*i*) Find i^6. Where i is imaginary unit.

(*ii*) Solve the equation $3x^2-2x+3=0$.

(*iii*) If $Z_1 = 2+i$ and $Z_2 = 2-i$, then find $\frac{Z_1}{Z_2}$.

(*iv*) For any complex numbers Z and W, prove that $\overline{Z+W} = \overline{Z}+\overline{W}$.