Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
100 level 2nd Semester Examination – Sept. / Oct. 2015
SCT102-2 Mathematics I

Uva Wellassa University

Instructions to candidates
Duration: 02 hours
Number of questions: 4 essay questions
Mark allocation: 100 mark
Scientific calculators are allowed
Answer all questions

1.

a. Using the definition, show that $\lim_{n\to\infty} x_n = 1/2$, where $x_n = \left(\frac{2n-1}{4n+1}\right)$. (6 mark)

b. If 1000 is invested at 6% compounded annual interst,then after n years total amount is a_n

i. Derive an expression for a_n. (5 mark)

ii. Find the first four terms of the sequence$\{a_n\}$. (8 mark)

iii. Is the sequence is convergent or divergent? Explain. (6 mark)

2.

a. Obtain the first four terms of each of the following sequences.

i. $\left\{\frac{(-1)^{n-1}}{2.4.6...2n}\right\}$ (5 mark)

ii. $\left\{\frac{\sqrt{n}}{2n+3}\right\}$ (5 mark)

iii. $\left\{\frac{(-1)^{n+1}}{2^n}\right\}$ (5 mark)

b. Using the **theorem of limits** evaluate the limits of each of the following sequences

i. $\lim_{n\to\infty} \frac{n^2+3}{2-n^2}$ (5 mark)

ii. $\lim_{x\to-\infty} \left(\frac{3x}{x-1} - \frac{2x}{x+1}\right)$ (5 mark)

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3.

a. Let $f = f(x, y, z)$,then define the partial derivative of f with respect to x *and* y. (2 mark)

b. Let $f(x, y) = 50x - 3x^2 + 30y - 2y^2 - 2xy - 20$,
Find f_x , f_y , f_{xx} , f_{yy} , f_{xy} , f_{yz} (12 mark)

c. A fish tank holds 5000 galloons of water, which drains from the bottom of the tank in 40 minutes, then the volume V of water remaining of the tank after t minutes is given below,

$$V(t) = 5000\left(1 - \frac{t}{40}\right)^2 \; ; 0 \leq t \leq 40$$

i. Find the rate at which the water drains from the tank at any time t. (4 mark)

ii. Find the draining rate at 5^{th} minute. (3 mark)

iii. At what time the flowing is fastest.(Hint:find $V'(t)$). (4 mark)

4.

a. Define the infinite series ? (2 mark)

b. Under what conditions does an alternating series converge? (2 mark)

c.

i. Test the convergence of the series;
$\sum_{n=1}^{\infty}|a_n| = \sum_{n=1}^{\infty}\frac{2^n}{(2n+1)!}$ where, $a_n = (-1)^{n+1}\frac{2^n}{(2n+1)!}$ (7 mark)

ii. Determine whether the series $\sum_{n=1}^{\infty}(-1)^n\frac{2n+1}{3n+2}$ is convergent or divergent. (7 mark)

iii. Show the series $\sum_{n=5}^{\infty}\frac{6+8n+9n^2}{3+2n+n^2}$ is divergent. (7 mark)