Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
300 Level 1st Semester Examination – May/July 2017
SCT 367-2 Mathematical Methods & Complex Analysis

Uva Wellassa University

Instructions to candidates
Duration: Two(02) hours
Number of questions: Six(06) Essay Questions
Mark allocation: 100 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer FOUR (04) questions ONLY

1.

a. Use the definition of Laplace transform to prove $L\{e^{at}\}=\frac{1}{s-a}$. (02 mark)

b. Determine the Laplace transform of the following functions.

i. $6t^3-3t^2+4t-2$ (02 mark)
ii. $2\cos 3t+5\sin 2t$ (02 mark)
iii. $4e^{2t}-\sinh 3t$ (02 mark)

c. Find $L^{-1}\{F(s)\}$ when $F(s)$ is given by:

i. $\frac{4s}{s^2+36}$ (02 mark)
ii. $\frac{3s-5}{s^2-25}$ (02 mark)
iii. $\frac{2s+6}{s^2+4}$ (02 mark)

d. State the **first shift theorem** and hence find $L\{e^{-2t}\sinh 3t\}$. (03 mark)

e. Determine the Laplace transform of $L\{t^2\cos 2t\}$. (03 mark)

f. Express $F(s)=\frac{5s-7}{(s+3)(s^2+2)}$ as partial fractions and hence determine its inverse transform.

(05 mark)

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2.

a. A function $f(t)$ is defined by,

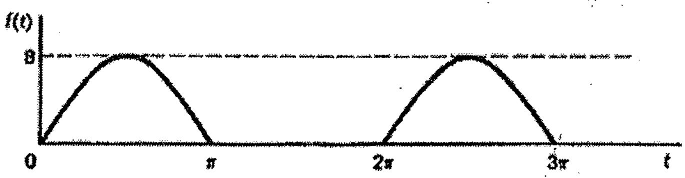
$$f(t)=\begin{cases} 3t^2 & \text{when } 0\le t\le 4 \\ 2t-3 & \text{when } 4<t<6 \\ 5 & \text{when } t>6 \end{cases}$$

Express $f(t)$ in terms *of Heaviside unit step functions* and obtain the Laplace transform of the function $f(t)$. (05 mark)

b. State the **second Shift theorem**. Hence, find $L^{-1}\left\{\frac{e^{-\pi s}(s+3)}{s(s^2+1)}\right\}$. (07 mark)

c. Determine the Laplace transform of the half-wave rectifier output waveform defined by,

$$f(t)=\begin{cases} 8\sin t & \text{when } 0<t<\pi \\ 0 & \text{when } \pi\le t<2\pi \end{cases}$$



(06 mark)

d. Determine $L^{-1}\left\{\frac{3(1-e^{-s})}{s(1-e^{-3s})}\right\}$ and sketch the resulting wave form of function. (07 mark)

3.

a. Solve the following ordinary differential equations by using Laplace transforms.

i. $\frac{dx}{dt}-4x=8$ at $t=0,\ x=2$. (04 mark)

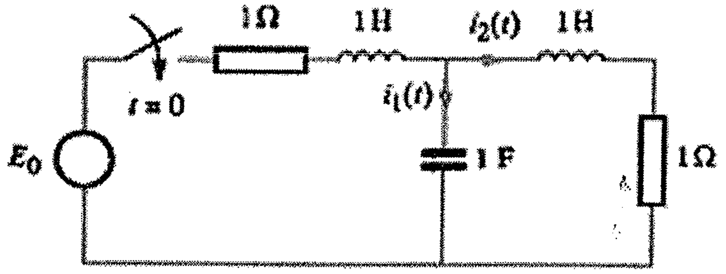
ii. $3\dot{x}-4x=\sin 2t$ at $t=0, x=\frac{1}{3}$. (04 mark)

iii. $\ddot{x}-2\dot{x}+5x=e^{2t}$ at $t=0, x=0, \dot{x}=1$. (07 mark)

b. Solve the following pair of simultaneous equations by using Laplace transform, where x and y are functions of t and given that at $t = 0,\ x = 0$ and $y = 0$. (10 mark)

$$\dot{y} + 3x = e^{-2t}$$
$$\dot{x} - 3y = e^{2t}$$

4. In the following circuit, there is no energy stored (that is, there is no charge on the capacitors and no current flowing in the inductances) prior to the closure of the switch at time $t = 0$. Use the Laplace transform to determine $i_1(t)$ for $t > 0$, for a constant applied voltage $E_0 = 10$ V. (25 mark)



5.

a. State the **Fourier's theorem.** (05 mark)

b. The clipped reponse of a half-wave rectifier is the periodic function, $f(t)$ of peroid 2π defined over the peroid $0 \le t \le 2\pi$ by,

$$f(t) = \begin{cases} 5\sin t & \text{when} \quad 0 \le t \le \pi \\ 0 & \text{when} \quad \pi < t \le 2\pi \end{cases}$$

Express $f(t)$ as a Fourier series expansion. (20 mark)

6.

a. State the **Cauchy-Riemann equations** for complex variable functions. (02 mark)

b. Find the *real* and *imaginary* parts of the function $f(z) = z^2 + 2z + 1$. Verify that they are analytic and find $f'(z)$. (06 mark)

c. Classify the **poles** and **zeros** of the following complex variable functions:

i. $\dfrac{2z+1}{z^2-z-2}$ (03 mark)

ii. $\dfrac{z+1}{(z-1)^2(z+3)}$ (03 mark)

iii. $\dfrac{3+4z}{z^3+3z^2+2z}$ (03 mark)

d. Determine the **residues** of the following functions at each pole in the finite z - plane:

i. $\dfrac{\cos z}{z}$ (03 mark)

ii. $\dfrac{1}{z^2(1-z)}$ (05 mark)