Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Science and Technology Degree Programme
1st year 1st Semester Examination – August / September 2014
SCT 103-0 Basic Calculus

Uva Wellassa University

Use standard symbols without definition.
Scientific calculators are allowed.
Number of questions: 04
Answer all questions
Time allocation: Two (02) hours
Mark allocation: 100 marks

1.

a. Find the following limits, if they exist:

i. $\lim_{x\to 1}(x^2 + 3x - 2)$ (5 Marks)

ii. $\lim_{x\to 2}\left(\frac{x^2+4x-12}{x^2-2x}\right)$ (5 Marks)

iii. $\lim_{x\to 0}\left(\frac{x^2+x}{x^3+x^2+x}\right)$ (5 Marks)

iv. $\lim_{x\to 3}\left(\frac{x^3-27}{x-3}\right)$ (5 Marks)

b. A rectangle has an area equal to 30 cm². Its width is equal to x. Find the perimeter of the rectangle as a function of x. (5 Marks)

2.

a. Differentiate following functions with respect to x.

i. $y = x^3 e^x$ (5 Marks)

ii. $y = \frac{\sqrt{x}}{1+x}$ (5 Marks)

iii. $y = 2\sqrt{x}\sin\sqrt{x}$ (5 Marks)

b. Find the rate of change of area of a circle with respect to its radius r. How fast does the area change with respect to radius when the radius is 8 cm?

(10 Marks)

3.

a. A man is moving away from a tower 40m high at a speed of 2 m/sec. Find the rate at which his angle of elevation of the top of the tower is changing when he is at a distance of 30 m from the foot of the tower. Assume that the height of the man is negligible compared to that of the tower. (10 Marks)

b. The fish population in a certain lake rises and falls according to the formula

$$F = 1000\,(30 + 17t - t^2)$$

F is the number of fish at time t, where t is measured in years since January 1, 2002, when the population of fish was first estimated.

i. Calculate the initial fish population in the lake. (2 Marks)

ii. On what date will the fish population again be the same as it was on January 1, 2002? (3 Marks)

iii. When will the fish population be maximum? (5 Marks)

iv. Find the maximum fish population in the lake. (5 Marks)

4.

a. Using integration, find the area of the region bounded by the line $2y + x = 8$, the x-axis and the lines x=2 and x=4. (10 Marks)

b. Integrate the followings.

i. $\int \left(5\,e^{-2x} + \frac{9}{x}\right) dx$ (5 Marks)

ii. $\int 3x^2 \sin x^3\, dx$ (5 Marks)

iii. $\int x^2 \sin x\, dx$ (5 Marks)

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