Uva Wellassa University, Sri Lanka
B.Tech. Degree Programme - 2006/07
BSc. (Computer Science) Degree Programme – 2006/07
End Semester Examination- Semester II
June/July - 2008

MAT 202-2 General Mathematics

Answer **all** questions
Time: Two (2) Hours

(1) (i) The transpose of a substance across a capillary wall in lung physiology has been modeled by the differential equation

$$\frac{dh}{dt} = -\frac{R}{V}\left(\frac{h}{k+h}\right)$$

Where h is the hormone concentration in the bloodstream, t is time, R is the maximum transport rate, V is the volume of the capillary, and k is a positive constant that measures the affinity between the hormones and the enzymes that assist the process. Solve the differential equation to find the relationship between hormone concentration and time.

(ii) Solve $\left(\frac{dy}{dx}\right)^2 = x^2\frac{d^2y}{dx^2}$ (Hint: take $\frac{dy}{dx} = p$)

(iii) One model for the spread of an epidemic is that the rate of spread is jointly proportional to the number of infected people and the number of uninfected people. In an isolated town of 5000 inhabitants, 160 people have a disease at the beginning of the week and 1200 have it at the end of the week. How long does it take for 80% of the population to become infected?

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(2) (i) Two tanks, tank I and tank II, are filled 1 m^3 of pure water. A solution containing 1 kg of salt per cubic meter is poured into tank I at a rate of 0.5 m^3 per minute. The solution leaves tank I at a rate of 0.5 m^3 per minute and enters tank II at the same rate. A drain is adjusted on tank II such that solution leaves tank II at a rate of 0.5 m^3 per minute.

(a) Show that after t minutes amount of salt at tank I is:

$$1 - e^{\left(\frac{-t}{2}\right)}$$

And amount of salt at tank II is:

$$\frac{1}{2}e^{\left(\frac{-t}{2}\right)}\left[2e^{\left(\frac{t}{2}\right)} - t - 2\right]$$

(b) Find salt amount in tank I and tank II after two minutes.

(c) What is the initial salt and eventual salt (salt when $t \to \infty$) in tank I?

(d) What is the initial salt and eventual salt in tank II? (Hint: use L' Hopitals' rule)

(e) What do you suggest to maintain 2 kg salt eventually in tank I and tank II? Explain briefly.

(ii) (a) In following pre-predator system, determine which of the variables, x or y represents the rabbits (prey) population and which represents the foxes (predator) population. Explain briefly.

$$\frac{dx}{dt} = -0.05x + 0.0001xy$$

$$\frac{dy}{dt} = 0.1y - 0.005xy$$

(b) Show that $\frac{dx}{-x(0.05-0.0001y)} = \frac{dy}{y(0.1-0.005x)}$

(c) Hence show that

$$x^{0.1}e^{-0.005x} = e^{D}y^{0.05}e^{0.0001y}$$

where D is an arbitrary constant

(3) (i) An intelligent agent knows that 60 aircraft, consisting of fighter planes and bombers, are stationed at a certain secret airfield. The agent wishes to determine how many of the 60 are fighter planes and how many are bombers. There is a type of rocket carried by both sorts of plains; the fighter carries six of these rockets, the bomber only two. The agent learns that 250 rockets are required to arm every plane at this airfield. Furthermore, the agent overhears a remark that there are twice as many fighter planes as bombers at the base. Calculate the number of fighter planes and bombers at the airfield or show that the agent's information must be incorrect because it is inconsistent.

(ii) Let the matrices

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 3 & 3 \\ 6 & 5 & 4 \end{pmatrix}_{3\times 3} \quad \text{and} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}_{3\times 2}$$

Then find,

(a) $A + 2I$

(b) AB

(c) $|A|$

(d) Hence deduce that whether inverse of A exists, if so find A^{-1}

(iii) Fruit Mart sells variety packs. The small pack contains three bananas, two apples, and one orange for Rs. 180. The medium pack contains four bananas, three apples, and three oranges for Rs. 305. The family size contains six bananas, five apples, and four oranges for Rs. 465. What price should Fruit Mart charge for special pack consists of one banana, one apple, and one orange?

(Hint: use part (d) of above question (ii))

(4) (i) Show that

(a)
$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$$

(b)
$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1+z \end{vmatrix} = xyz$$

(c) For what value of α does the following matrix not have an inverse?

$$\begin{pmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{pmatrix}$$

(ii) A certain electric company produces two types of motors, each on a separate assembly line. The respective daily capacities of the two lines are 600 and 750 motors. A type I motor uses 10 units of a certain electric component, and a type II motor uses only 8 units. The supplier of the component can provide 8000 pieces a day. The profits per motor for types I and II are Rs. 600 and Rs. 400 respectively.

(a) Derive linear programming model.

(b) Find optimal daily production of type I and II that maximize profit.

(c) What is the maximum profit?

(d) Determine the range of the ratio of unit profits that will keep the solution in part (b) unchanged.

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