

Uva Wellassa University, Sri Lanka.
End Semester Examination - September/October 2012
SCT 302-2 Advanced Mathematics II



Time: Two (2) hours

Answer all (04) questions.

Calculators are allowed.

01. a.) For the following matrix determine the eigenvectors corresponding to each eigenvalue and determine a basis for the eigenspace of the matrix corresponding to each eigenvalue.

$$A = \begin{pmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{pmatrix}$$

b.) Let $A = \begin{pmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{pmatrix}$. Show that $A^3 + (a^2 + b^2 + c^2)A = 0$.

(25 Marks)

02. a.) Determine if the transformation $T: R^2 \rightarrow R^3$ defined as $T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \\ 3y \end{pmatrix}$ is linear or

not.

- b.) Determine the new point after applying the transformation to the given point

i.) $x = (2, -4, 1)$ reflected about the xz -plane.

ii.) $x = (10, 7, -9)$ projected on the x -axis.

iii.) Rotate $x = (4, 2)$ 45° counter-clockwise and then project on the x -axis.

(25 Marks)

03. a.) i.) Find the function $P(x) = a + b \cos(\pi x) + c \sin(\pi x)$, which interpolates the data

x	0	0.5	1
y	2	5	4

ii.) Find the quadratic polynomial interpolating this data.

b.) Find the cubic spline function for the following data points using natural boundary conditions.

x	0	1	2	3
y	2.5	0.5	0.5	1.5

(25 Marks)

04. a.) Do three iterations of the bisection method applied to $f(x) = x^3 - 2$ using initial interval $[a, b] = [0, 2]$

b.) Do three iterations of Euler's method for the following initial value problem letting $h=0.2$

$$\frac{dy}{dt} = t + y ; y(0) = 1 ; 0 \leq t \leq 1$$

(25 Marks)