

Instructions to candidates

Duration: Two(02) hours

Number of questions: Four(04) essay questions

Mark allocation: 100 mark

Use standard symbols without definition.

Scientific calculators are allowed.

Answer all questions



1.

a. Use **L' Hospital's rule** to evaluate following limits.

i. $\lim_{x \rightarrow 1} \frac{x^3 + x^2 - x - 1}{x^2 + 2x - 3}$ (10 mark)

ii. $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^2}$ (10 mark)

b. Find $\lim_{x \rightarrow 1} f(x)$, where:

$$f(x) = \begin{cases} 4 - x, & x \leq 1 \\ 4x - x^2, & x > 1 \end{cases}$$

(04 mark)

c. Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$, then show that f is continuous at 0 or not. (08 mark)

2.

a. Obtain the first four terms of each of the following sequences.

i. $\{n^2\}_{n=1}^{\infty}$ (05 mark)

ii. $\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty}$ (05 mark)

iii. $\{3 + (-1)^n\}_{n=1}^{\infty}$ (05 mark)

b. From the $\epsilon - N$ definition, show that $\lim_{n \rightarrow \infty} \frac{1}{n^2} = 0$. (05 mark)

- c. Prove that the sequence $\left\{ \frac{n+1}{2n} \right\}_{n=1}^{\infty}$ converges to $\frac{1}{2}$. (05 mark)

3.

- a. Determine whether each of the following series is **convergent or divergent**.

i. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ (05 mark)

ii. $1 + 4 + 16 + 64 + \dots$ (02 mark)

- b. If $f(x, y) = x^2 + y^2 - 2xy$, then find f_x , f_y , f_{xx} and f_{yy} . (08 mark)

4.

- a. Determine a truth value of the following statements.

i. "2 is a prime number" or "4 > 1" (02 mark)

ii. "2 is a prime number" and "4 > 1" (02 mark)

iii. If "2 + 2 = 4" then "3 - 1 = 4" (02 mark)

iv. If "2⁰ = 1" only if "e⁰ = 1" (02 mark)

- b. Construct a truth table for the following statements:

i. $P \rightarrow (P \wedge Q)$ (10 mark)

ii. $P \rightarrow (P \vee Q)$ (10 mark)