Uva Wellassa University

Time: Two (02) hours

Part C- Essay Questions

Answer all questions.

Marks allocation: 50 Marks

---

01. A machine is set to produce nails of length 10cm, with standard deviation 0.05cm. The lengths of the nails are normally distributed.

a) Find the percentage of nails produced between 9.95cm and 10.08cm in length.

(05 Marks)

b) The machine's setting is moved by a careless worker with the consequence that 30% of the nails are under 8.9cm in length and 16% are over 9.2cm. Find the new mean and standard deviation of the length of nail. (05 Marks)

02. n = 100 random samples of water from the Badulu Oya were taken and the calcium concentration (milligrams per liter) measured. A 95% Confidence Interval on the mean calcium concentration is $0.49 \leq \mu \leq 0.82$.

a) Would a 99% confidence interval calculated from the same sample data been longer or shorter? (02 Marks)

b) Consider the following statement: There is a 95% chance that μ is between 0.49 and 0.82. Is this statement correct? Explain your answer. (03 Marks)

03. Consider a situation where both Y (Response variable) and X (Predictor variable) are random variables. Let S_x and S_y be the sample standard deviations of the observed x's and y's, respectively. Show that an alternative expression for the fitted simple linear regression model $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ is $\hat{y} = \bar{y} + r\frac{S_y}{S_x}(x - \bar{x})$. (10 Marks)

You may use the following,

Sample correlation coefficient: $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

OLE estimators: $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$, and $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$

04. The grams of solids removed from a material (y) is thought to be related to the drying time(x). Ten observations obtained from an experimental study follow:

#	1	2	3	4	5	6	7	8	9	10
y	4.3	1.5	1.8	4.9	4.2	4.8	5.8	6.2	7.0	7.9
x	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0

Summary statistics:

$$\sum_{i=1}^{10} x_1 = 47.5 \qquad \sum_{i=1}^{10} y_1 = 48.4$$

$$\sum_{i=1}^{10} x_i^2 = 246.25 \qquad \sum_{i=1}^{10} y_i^2 = 272.16$$

$$\sum_{i=1}^{10} x_i y_i = 253.95$$

a) Construct a scatter diagram for these data and comment on it. (02 Marks)

b) Estimate the correlation coefficient and test the following hypotheses.

$H_0: \rho = 0.80$ Vs. $H_0: \rho > 0.80$ (04 Marks)

c) Fit a simple linear regression model. (04 Marks)

d) Construct the ANOVA table. (05 Marks)

e) Test for significance of regression by using $\alpha = 0.05$. What conclusions can you draw? (01 Mark)

f) Based on these data, what is your estimate of the mean grams of solids removed at 4.25 hours? (02 Marks)

g) Compute the predicted values($\hat{y}_i$). (02 Marks)

h) Compute the residuals ($\hat{\varepsilon}_i$). (02 Marks)

i) Plot the residuals ($\hat{\varepsilon}_i$) versus predicted values ($\hat{y}_i$) and comment on the underlying regression assumptions. Specifically, does it seem that the equality of variance assumption is satisfied? (03 Marks)

-End-