Solving Poisson's equation with Dirichlet boundary condition using Henstock-Kurzweil integral

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The Poisson's equation is an elliptic type second order partial differential equation, which has several applications in theoretical physics, chemistry and engineering. There are a few approaches to solve the Poisson's equation such as the Green's function, Dirichlet's principle, layer potentials, L^2 estimates, energy methods, etc. However, in most cases it is difficult to find an analytical solution. HK-integration has more powerful results such as: convergence theorem, Fundamental theorem of calculus with full generality, integration over unbounded region, integral containing parameters, etc. It would be a good idea to find an analytical solution to Poisson's equation with Dirichlet boundary condition for general setting with more advanced integral like Henstock—Kurzweil integral (HK-integral). We use the Green's function to obtain a general representation formula for the solution of the Poisson's equation. For this, Green's function G needs to exist. Moreover, it is difficult to calculate an explicit formula for G. Thus, we need a new technique or method to evaluate. So we use HK.-integration to find G and a representation formula for the solution. HK-integration is very simple to describe as the Riemann integral, which possesses all the advantages of the Lebesgue integral and even more. Moreover, any Riemann or Lebesgue integrable function is HK-integrable. Therefore, the set of all HK-integrable functions is larger than those of the set of other integrals. So, using HK-integral, one can find an analytical solution to the Poisson's equation with Dirichlet boundary condition, where the case Riemann or Lebesgue integration does not work.

Keywords: Poisson's equation, Dirichlet boundary condition, Henstock-Kurzweil integral Green's function