UvaWellassa University, Sri Lanka
Faculty of Science and Technology
Industrial Information Technology Degree program
1st Semester Examination – March/ April 2013

Uva Wellassa University

IIT 372-2 Operational Research

Instructions to candidates
Answer all questions.
Number of Questions: Four (04).
Time allocation: Two (02) hours.
Total marks allocated: 100.
Calculators are allowed .

1.

a. A company manufactures two products P_1 and P_2. These products are processed on two machines M_1 and M_2. The time required to manufacture one unit of each of the two products and capacities of each machines are given below.

Machine	Time Required (minutes)		Machine capacity (minutes)
	P_1	P_2	
M_1	15	20	100
M_2	20	25	130

The product P_1 sells at a profit of Rs. 100/= per unit, while P_2 sells at Rs. 200/= per unit. The company needs to determine the quantities of P_1 and P_2 to be manufactured in order to maximize the total profit earned, subject to the constraints on machine capacities.

Formulate the above problem as a Linear Programming (LP) Model.

(10 marks)

b. UWUB Ltd produces two types of boots B_1 and B_2. A pair of B_1 requires two man-hours for cutting, 1 man-hour for sewing and 3 man-hours for finishing while a pair of B_2 requires 2 man-hours for cutting, 2 man-hours for sewing and 1 man-hour for finishing. Each day the company has available 140 man-hours for cutting, 120 man-hours for sewing and 150 man-hours for finishing. Each pair of B_1 contributes Rs. 100/= to the total profit and B_2 contributes Rs. 80/=. The company wishes to determine the optimal

35

product-mix in order to maximize the profit. The formulated linear programming model is given below.

$$\begin{array}{llllllll} \textit{Maximize} & P & = & 100X_1 & + & 80X_2 & & \\ \textit{Subject to} & & & & & & & \\ & 2X_1 & + & 2X_2 & \leq & 140 & & (\textit{Cutting}) \\ & X_1 & + & 2X_2 & \leq & 120 & & (\textit{Sewing}) \\ & 3X_1 & + & X_2 & \leq & 150 & & (\textit{finishing}) \\ & & X_1, & X_2 & \geq & 0 & & \end{array}$$

i. Use the graphical method to solve the above formulated LP model

ii. Compute the total profit of the optimal solution

iii. If the capacity in sewing is increased from 120 man-hours to 150 man-hours, will the optimal solution be affected? Justify your answer.

(20 marks)

2. The Primal of a Linear Programming Problem (LPP) is given bellow.

$$\begin{array}{lllllllll} \textit{Minimize} & Z & = & 10X_1 & + & 12X_2 & + & 15X_3 & \\ \textit{Subject to} & & & & & & & & \\ & 6X_1 & + & 7X_2 & + & 8X_3 & \geq & 100 & \\ & 7X_1 & + & 6X_2 & + & 9X_3 & \geq & 170 & \\ & 8X_1 & + & 10X_2 & + & 6X_3 & \geq & 150 & \\ & & X_1, & X_2, & X_3 & & \geq & 0 & \end{array}$$

a. Obtain the Dual problem of the given primal LPP

b. Construct the initial Simplex tableau for the dual problem

c. Perform the FIRST Simplex iteration (Second solution). Is this the optimal solution?

(20 marks)

3. A manufacturer produces four types of products (P_1, P_2, P_3 and P_4) using three resources (labour, material and machine time). The resource required for each product with the profit per unit sold is given in Table 3.1. The variable X_i (i =1, 2, 3, 4) represents the number of units of product i to be manufactured,

Product Type	labour	material	machine time	Profit (Rs.)
P_1	1	5	1	3
P_2	1	3	3	3
P_3	1	2	5	4
P_4	1	1	8	7

Table 3.1

The company estimate that, for the next day, they have 9 hours of labour hours, 60 kg of raw material and 50 machine hours. The company wants to know how many of each type should the company produce in the next day, to get a maximum profit. The formulated Linear Programming Problem (LPP) model is given below.

Minimize $Z_{PROFIT} = 3X_1 + 3X_2 + 4X_3 + 7X_4$

Subject to

$$X_1 + X_2 + X_3 + X_4 \leq 9 \quad \text{(Labour)}$$
$$5X_1 + 3X_2 + 2X_3 + X_4 \leq 60 \quad \text{(Raw Material)}$$
$$X_1 + 3X_2 + 5X_3 + 8X_4 \leq 50 \quad \text{(Machine)}$$
$$X_1, X_2, X_3, X_4, \geq 0$$

(a).

(i) Obtain the standard form of the above model.

(ii) Construct the initial simplex tableau of the simplex method (You are not required solve problem).

35

(b). The above problem was solved using the simplex method and the final tableau is given in the **Table 3.2**.

	C_i	Basic variables	3 X_1	3 X_2	4 X_3	7 X_4	0 S_1	0 S_2	0 S_3	Solution Value
Row1	3	X_1	1	5/7	3/7	0	8/7	0	-1/7	22/7
Row2	0	S_2	0	-6/7	-5/7	0	-39/7	1	4/7	269/7
Row3	7	X_4	0	2/7	4/7	1	-1/7	0	1/7	41/7
		Z_i	3	29/7	37/7	7	17/7	0	4/7	353/7
		$C_i - Z_i$	0	-8/7	-9/7	0	-17/7	0	-4/7	

Table 3.2

i. Define S_1, S_2, and S_3

ii. Determine the optimal product mix.

iii. What is the maximum profit?

iv. Is there any binding resources?

v. What are the shadow prices associated with the resources? Interpret these shadow prices

(30 marks)

4. ABC Ltd operates 3 factories (F_1, F_2, and F_3) where a common product is manufactured. The product is distributed to 3 markets M_1, M_2, and M_3. The monthly capacities of the 3 factories are 27, 9, and 12 units of product respectively, and monthly market demand are 15, 18 and 15 units. The per unit transport costs from each factory to each market are given in the table.

	F_1	F_2	F_3
M_1	Rs.18/=	Rs.12/=	Rs.8/=
M_2	Rs.8/=	Rs.20/=	Rs.4/=
M_3	Rs.30/=	Rs.14/=	Rs.20/=

a. Formulate the above Linear programming problem as Balance Transportation Problem

b. Use North-West Corner Rule method to obtain the initial feasible solution

c. Use Stepping Stone Method to determine whether the current solution (in **part (b)**) is the optimal (you are not required solve the problem)

(20 marks)