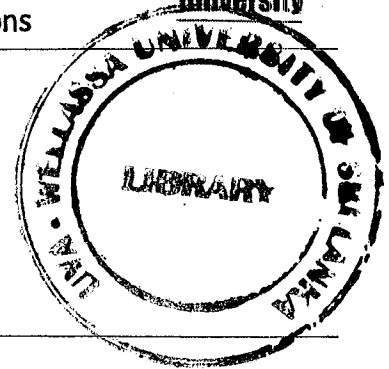
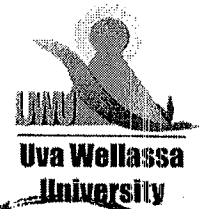


Uva Wellassa University of Sri Lanka  
Faculty of Science and Technology  
Department of Science and Technology  
200 Level 2<sup>nd</sup> Semester Examination – Dec./Jan. 2016/17  
SCT 202-3 Differential Equations and Applications



**Instructions to candidates**

**Duration:** Two (02) hours

**Number of questions:** Four (04) Essay Questions

**Mark allocation:** 100 mark

Use standard symbols without definition.

Scientific calculators are allowed.

**Answer all questions**

**PART A**

1.

a. Let  $y = x \sin \frac{1}{x}$  for  $x \neq 0$ . Show that  $x \frac{dy}{dx} = y - \cos \frac{1}{x}$ . (04 mark)

b. Find the general solution of  $xy \frac{dy}{dx} = \frac{x^2 + 1}{y + 1}$  by *separating the variables*. (04 mark)

c. Solve the homogeneous equation  $(2y - x) \frac{dy}{dx} = (2y + x)$  by substituting  $y = vx$ . (06 mark)

d. Determine the general solution of the equation  $x \frac{dy}{dx} - y = x^3 + 3x^2 - 2x$  by applying *integrating factor method*. (06 mark)

2. Solve the following second order differential equations.

a.  $\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 4y = 0$  (04 mark)

b.  $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 4y = 0$  (06 mark)

c.  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$  (10 mark)

d.  $\frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + 10y = 2 \sin 2x$  (10 mark)

3. Determine the general solution of the following system of equations.

a.  $\dot{x} = -3x + \sqrt{2}y$   
 $\dot{y} = \sqrt{2}x - 2y$

(15 mark)

b.  $\dot{x} = x - y$   
 $\dot{y} = x + 3y$

(15 mark)

4. In each of the following cases, find  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$ ,  $\frac{\partial^2 z}{\partial x^2}$ ,  $\frac{\partial^2 z}{\partial y^2}$  and  $\frac{\partial^2 z}{\partial x \partial y}$ .

a.  $z = 3x^2 + 2xy + 4y^2$

(06 mark)

b.  $z = \sin xy$

(06 mark)

c.  $z = (x + y)\ln(xy)$

(08 mark)