Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
200 Level 2nd Semester Examination – Dec./Jan. 2016/17
SCT 202-3 Differential Equations and Applications

Instructions to candidates
Duration: Two (02) hours
Number of questions: Four (04) Essay Questions
Mark allocation: 100 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions

PART A

1.

a. Let $y = x\sin\frac{1}{x}$ for $x \neq 0$. Show that $x\frac{dy}{dx} = y - \cos\frac{1}{x}$. (04 mark)

b. Find the general solution of $xy\frac{dy}{dx} = \frac{x^2+1}{y+1}$ by *separating the variables*. (04 mark)

c. Solve the homogeneous equation $(2y-x)\frac{dy}{dx} = (2y+x)$ by substituting $y = vx$. (06 mark)

d. Determine the general solution of the equation $x\frac{dy}{dx} - y = x^3 + 3x^2 - 2x$ by applying *integrating factor method*. (06 mark)

2. Solve the following second order differential equations.

a. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 0$ (04 mark)

b. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ (06 mark)

c. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$ (10 mark)

d. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 10y = 2\sin 2x$ (10 mark)

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3. Determine the general solution of the following system of equations.

a. $\dot{x} = -3x + \sqrt{2}y$
$\dot{y} = \sqrt{2}x - 2y$ (15 mark)

b. $\dot{x} = x - y$
$\dot{y} = x + 3y$ (15 mark)

4. In each of the following cases, find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}$ and $\frac{\partial^2 z}{\partial x \partial y}$.

a. $z = 3x^2 + 2xy + 4y^2$ (06 mark)

b. $z = \sin xy$ (06 mark)

c. $z = (x + y)\ln(xy)$ (08 mark)