Uva Wellassa University, Sri Lanka

End Semester Examination – February / March 2012

CST 475-2 Graph Theory

Duration: Two (02) hours

Uva Wellassa University

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Total **four (04)** questions.

Answer **all** questions.

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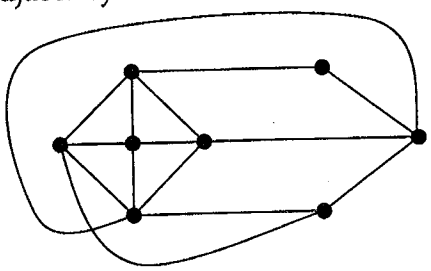
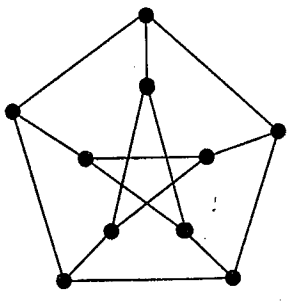
01. (a) Write down the adjacency matrix and incidence matrix for the following graph G:

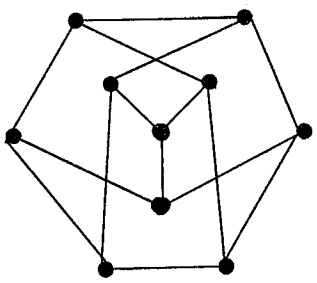


G

(b) Determine whether the following pair of graphs is isomorphic:

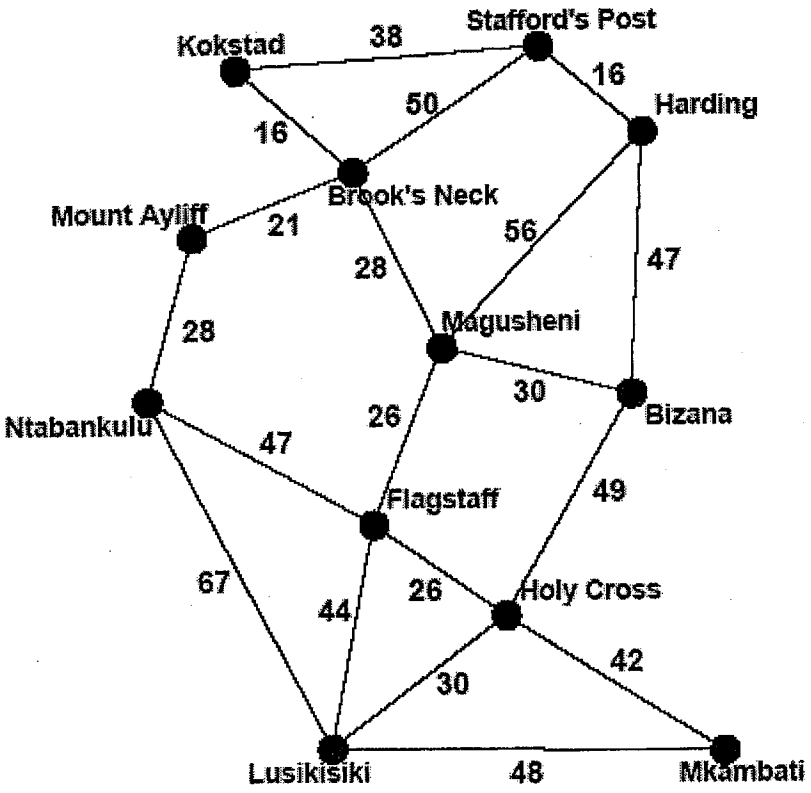


H

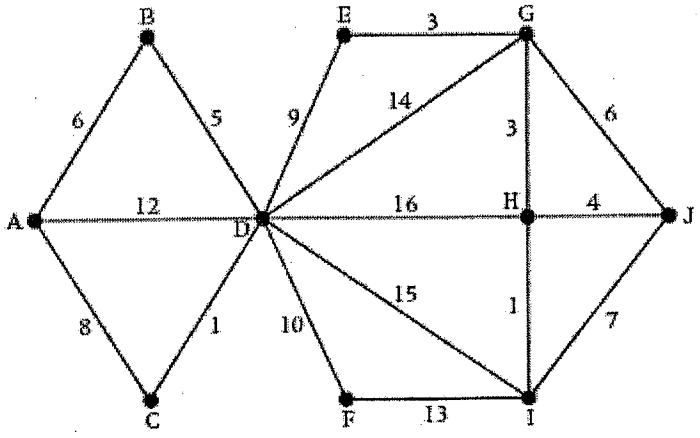


$H^*$

(c) Find the shortest path between Kokstad and Mkambati in the following graph:




(d) A theme park employs a student to patrol the paths and collect litter. The paths that she has to patrol are shown in the following diagram, where all distances are in meters.

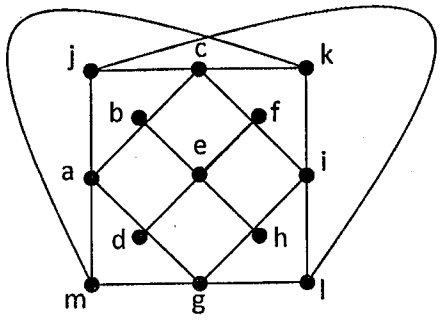



i. Find an optimal Chinese postman route that the student should take if she is to start and finish at A.

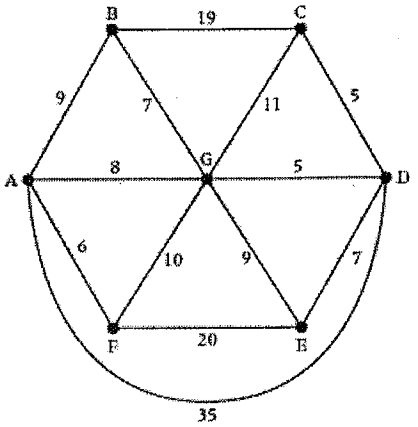
ii. State the length of your route.

02. (a) i. Define the term *minimal spanning tree* for a connected graph.

ii. Perform a breath first search and a depth first search for the following graph:

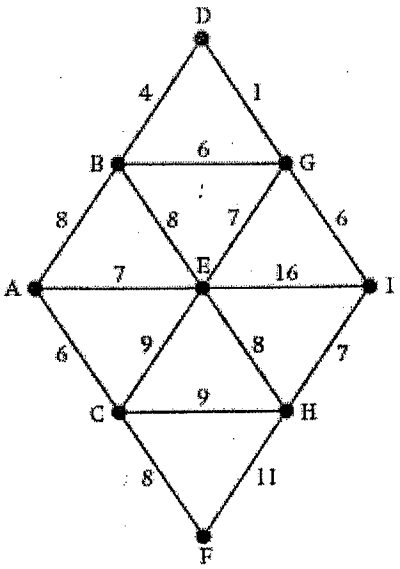



(b) Let G be the weighted graph given below. Find a minimum weighted spanning tree T of G using Kruskal's algorithm.




G

(c) Find the minimum weighted spanning tree for the graph given below using Prime's algorithm:



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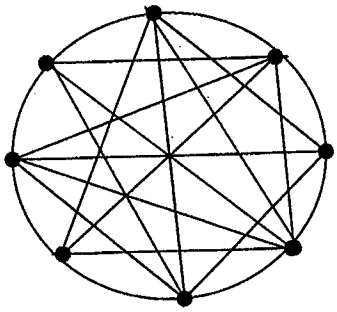
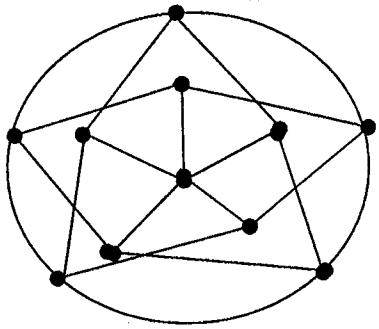
03. (a) i. Let G be a simple graph. Define k-coloring of G and the chromatic number of G.

ii. Find the chromatic number for the following graphs:

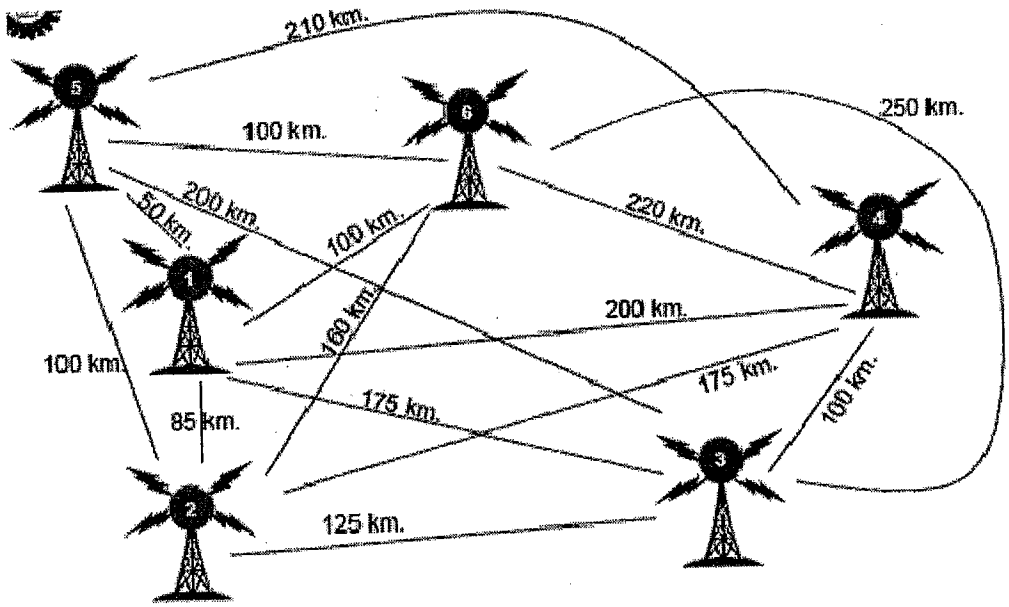


G



H

(b) Use graph coloring to solve the following problem:

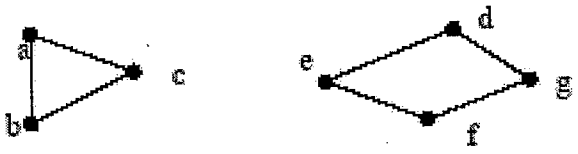



Two radio stations cannot use the same frequency channel when they are within 150 km of each other. How many channels are needed?

04. (a) i. Define an isomorphism of two graphs.

ii. Draw all the non-isomorphic simple connected graphs with four vertices.

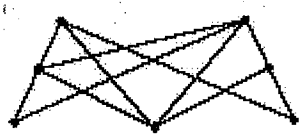
iii. Let H be the following disconnected graph. Draw its dual $H^*$ and the dual of the dual $(H^*)^*$



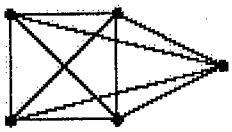

H

iv. Show that if G is a disconnected planar graph, then $G^*$ is connected.

(b) Define the term *planar graph*. Determine whether each of the following graphs is planar or not. Justify your answers



(i)



(ii)