Uva Wellassa University, Sri Lanka
End Semester Examination – August 2011
SCT 102-2 / SCT 105-2 Mathematics I

Time: Two (02) hours

Answer **All (04)** questions.
Total **three (03)** pages.

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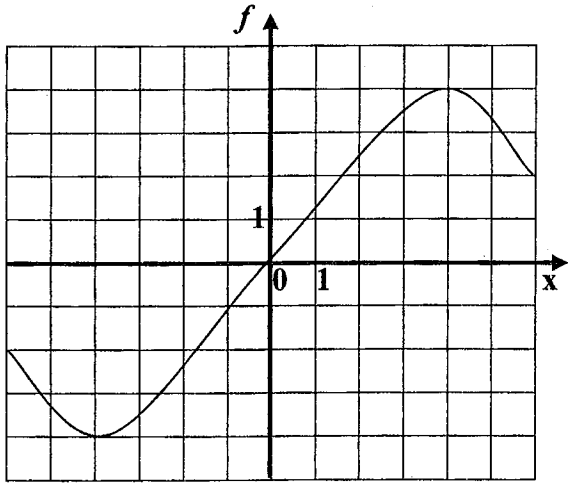
01. a.) Let f be the function whose graph is given below.



i.) State the value of $f(4)$.

ii.) For which values of x, $f(x)$ is -4.

iii.) State the domain of f.

iv.) State the range of f.

v.) On which interval is f increasing.

(10 marks)

b.) The population of a certain species in a limited environment with initial population 100 and carrying capacity 1000 is given by

$$P(t) = \frac{100{,}000}{100 + 900e^{-t}}$$ where t is measured in years.

i.) Estimate how long it takes for the population to reach 900.

ii.) Find the inverse of this function.

iii.) Use the inverse function to find the time required for the population to reach 900.

(10 marks)

c.) Find the constant c that makes f continuous on $(-\infty,\infty)$.

$$f(x)=\begin{cases} x^2-c^2 & \text{if } x<4 \\ cx+20 & \text{if } x\geq 4 \end{cases}$$

(05 marks)

02. a.) i.) State the Intermediate Value Theorem.

ii.) If $f(x)=x^3-x^2+x$, show that there is a number c such that $f(c)=10$.

iii.) Determine all the numbers c which satisfies the conclusions of the Mean Value Theorem for the following function.

$$f(x)=x^3+2x^2-x \quad ; \quad on\ [-1,2]$$

(10 marks)

b.) If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is $v=\frac{mg}{c}\left(1-e^{-ct/m}\right)$

where g is the acceleration due to gravity and c is a positive constant.

i.) Calculate $\lim_{t\to\infty} v$

ii.) For fixed t, use l'Hospital's Rule to calculate $\lim_{m\to\infty} v$. What can you conclude about the speed of a very heavy falling object ?

(15 marks)

03. a.) i.) What is a sequence?

ii.) What does it mean to say $\lim_{n\to\infty} a_n = 4$?

iii.) What does it mean to say $\lim_{n\to\infty} a_n = \infty$?

(06 marks)

b.) If Rs. 10,000 is invested at 6.5% interest, compounded annually, then after n years the investment is worth $a_n = 10000\left(1+\frac{6.5}{100}\right)^n$ rupees.

i.) Find the first four terms of the sequence.

ii.) Is the sequence convergent or divergent? Explain.

(06 marks)

c.) i.) What is an alternating series?

ii.) Under what conditions does an alternating series converge?

iii.) Test the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{n^3+4}$ for the convergence or divergence.

(08 marks)

04. a.) Determine if the integral $\int_{-2}^{2}\frac{1}{x^4}dx$ is convergent or divergent. If it is convergent find its value.

(10 marks)

b.) The average speed of molecules in an ideal gas is

$$\bar{v} = \frac{4}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2}\int_0^{\infty} v^3 e^{-Mv^2/2RT}\,dv$$

Where, M = molecular weight

R = gas constant

T = gas temperature

v = molecular speed

Show that $\bar{v} = \sqrt{\frac{8RT}{\pi M}}$

(20 marks)