Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Science and Technology
300 Level 2nd Semester Examination – Jan./Feb. 2016
SCT 302-2 Advanced Mathematics II

Uva Wellassa University

Instructions to candidates
Duration: Two(02) hours
Number of questions: Four(04) essay questions
Mark allocation: 100 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions

1.

a. Mr. Silva is the production manager for a manufacturing company. The company produces tables (x) and chairs (y). Each table generates a profit of Rs.80 and requires 3 hours of assembly time and 4 hours of finishing time. Each chair generates Rs.50 of profit and requires 3 hours of assembly time and 2 hours of finishing time. There are 360 hours of assembly time and 240 hours of finishing time available each month.

i. Formulate this as a linear programming problem. (05 mark)
ii. Find the optimal solution using **graphical method**. (10 mark)

b. Solve the following LP problem using **simplex method**.

$$minimize\ z = x_1 - 3x_2 + 2x_3$$

subject to

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$ (15 mark)

2.

a. A root of the equation $x^3 + 3x^2 - 3 = 0$ lies between -3 and -2. Find this root, with an accuracy 10^{-3}, by **interval bisection method.** (10 mark)

b. Using **Newton-Raphson method**, find a real root, with an accuracy 10^{-3}, of the equation $\sin x - \frac{x}{2} = 0$ given that the root lies between $\pi/2$ and π. (Take $x_0 = \frac{\pi}{2}$) (10 mark)

3.

a. Solve the following system of equations by **Gauss-Jordan method.**

$$\begin{aligned} 3x + 2y + 4z &= 7 \\ 2x + y + z &= 7 \\ x + 3y + 5z &= 2 \end{aligned}$$

(15 mark)

b. Use the **Jacobi's method** to solve the following linear system;

$$10x_1 + x_2 + x_3 = 12$$

$$x_1 + 10x_2 + x_3 = 12$$

$$x_1 + x_2 + 10x_3 = 12$$

where $x_1^{(0)} = 0.4$, $x_2^{(0)} = 0.6$ and $x_3^{(0)} = 0.8$. (10 mark)

4.

a. Use **forward difference method** and **backward difference method** to approximate $f'(0.5)$, where $f(x) = x\cos x$ with $h = 0.1$. (10 mark)

b. Evaluate following integral;

$$I = \int_0^1 \frac{1}{1+x^2}\,dx$$

using both **mid point rule** and **Trapezoidal rule.** (15 mark)

Useful Formulas

1. Newton-Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

2. Jacobi's Method

$$x_i^{k+1} = a_{ii}^{-1}\left(b_i - \sum_{m=1}^{n} a_{im}x_m^k\right) \text{for all } i = 1,2,3,\ldots\ldots,\text{n}; m \neq i$$

3. Forward Difference Method

$$f'(x_i) = \frac{f(x_i + h) - f(x_i)}{h}$$

4. Backward Difference Method

$$f'(x_i) = \frac{f(x_i) - f(x_i - h)}{h}$$

5. Mid Point Rule

$$\int_a^b f(x)dx = (b-a)f\left(\frac{a+b}{2}\right)$$

6. Trapezoidal Rule

$$\int_a^b f(x)dx = (b-a)\left(\frac{f(a)+f(b)}{2}\right)$$