Uva Wellassa University

Faculty of Management

Degree of Bachelor of Business Management in Entrepreneurship and Management

3rd YEAR 1st SEMESTER EXAMINATION – FEBRUARY/MARCH 2011

EMG 333-2 Actuarial Statistics I

Uva Wellassa University

Instructions to candidates

No. of pages	: Two (02)
No. of questions	: Three (03) Essay
Time	: One (01) Hour
Marks allocated	: 50 Marks

Answer all questions

Part C: Essay Questions

1. Consider 2 classes of 45 year olds under a health insurance plan. The nonsmoker class has 750 insured with annual claims following a uniform distribution between Rs. 200 and Rs. 2600. The smoker class has 125 insured with annual claims following a uniform distribution between the Rs. 500 and Rs. 4500. What is the probability that the difference between the average smoker claims and the average nonsmoker claims in under Rs. 1000?

(10 Marks)

2. The claim amounts of an insurance company are known to be distributed as an exponential with positive parameter $\boldsymbol{\theta}$. The probability density function (p.d.f) of the exponential distribution is

$$f_X(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \quad \text{where } x > 0, \text{and } \theta > 0$$

a) Verify that this is indeed a probability density function.

b) Show that the cumulative distributions function of $f_X(x)$ is $F_X(x) = 1 - e^{-x/\theta}$ when $x > 0$.

c) Using CDF of X, show that the median claim amount of the company is given as $\mathbf{m = \theta \ln 2}$.

d) The Moment generating function of X, $M_X(t)$ is given by $M_X(t) = \left(\frac{1}{1-\theta t}\right)$, where $t > \frac{1}{\theta}$. Using the MGF of X

i. Show that the mean of the claim amounts is $E[X] = \theta$

ii. Show that the variance of claim amounts of the company is $V[X] = \theta^2$

10

Note:

$$E[X^r] = \mu_r' = \frac{d^r}{dt^r} M_X(0)$$

$$E[X^2] = \mu_2' = \frac{d^2}{dt^2} M_X(t)\Big|_{t=0}$$

(20 Marks)

3.

a) A certain insurance policy introduced to the market in year 2010. It was found that the number of claim, X has a probability density function of the form:

$$f_X(x) = \begin{cases} c(5-x) & for\ x = 0,1,2,3,4 \\ 0 & elsewhere \end{cases}$$

i. Find the constant c.

ii. Find the cumulative distribution function, $F_X(x)$ and sketch the graph of the CDF.

iii. Find the probability, $P(0 < X \leq 2)$.

b) The number of claims occurs in during the year distributed as a Poisson with the mean λ. The probability mass function of the Poisson distribution is

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ where } \lambda > 0 \text{ and } x = 0,1,2,\dots$$

The Skewness of a distribution is defined to be $\gamma_1 = \frac{\mu_3}{\sqrt{\mu_2{}^3}}$

where $\mu_r = E[(X-\mu)^k]$

$\mu = E[X]$

Show that $\gamma_1 = \frac{1}{\sqrt{\lambda}}$ for the Poisson distribution.

(20 Marks)

Note:

For the given Poisson distribution with mean λ, following results can be used without proving.

$E[X] = \lambda$

$E[X^2] = \lambda + \lambda^2$

$E[X^3] = \lambda^3 + 3\lambda^2 + \lambda$

$\mu_3 = E[(X-\mu)^3] = E[X^3] - 3\lambda E[X^2] + 3\lambda^2 E[X] - \lambda^3$

$\mu_2 = E[(X-\mu)^2] = E[X^2] - [E(X)]^2$