Uva Wellassa University, Sri Lanka
End Semester Examination – February/March 2012
SCT 103-0 Basic Calculus

Duration: Two (02) hours

Total four (04) Questions
Answer all questions

01. a.) Factor each of the following:

i. $18x^3 - 8x$

ii. $x^2 - 2y + y^2 - 4$

iii. $3x^4 - 5x^2 + 2$

iv. $4m^3n - 2m^2n^2 + 2mn^3$

b.) Simplify the followings:

i. $\frac{x^2-6x+9}{x^2-9}, x \neq \pm 3$

ii. $\frac{10x^3y}{3xy+9y} \cdot \frac{x^2-9}{4x^2-12x}$

iii. $\frac{2x^3-2x^2y+2xy^2}{x^3y-xy^3} \div \frac{x^3+y^3}{x^2+2xy+y^2}$

iv. $\frac{x+3}{x^2-6x+9} - \frac{x+2}{x^2-9} - \frac{5}{3-x}$

c.) Solve the following simultaneous equations using matrices or determinants:

i. $y - 3x = 5 ; y - 4x = 7$

ii. $4x + 5y = -8 ; 3x + y = 5$

iii. $\frac{6}{5}u + \frac{3}{2}v = -4 ; \frac{1}{3}u - \frac{5}{4}v = 1$

02. a.) Define the terms domain and range of a function.

b.) Determine whether each of the followings is a function:

i. $y = 5x$

ii. $y^2 + x^2 - 2x = 1$

c.) Given $f(x) = \frac{x}{x^2+1}$, find each of the followings:

i. $f(0)$

ii. $f(-1)$

iii. $f(x + 1)$

iv. $f\left(-\frac{1}{2}\right)$

d.) Find the domain and the range of the following functions. Use interval notation whe it is applicable.

i. $f(x) = 5 - x$ ii. $g(x) = \sqrt{1 + x^2}$ iii. $h(x) = |x - 1|$

iv. $p(x) = \frac{7}{2x+1}$ v. $q(x) = \frac{1}{2}(x^2 + 3)$

03. a) Find each of the following limits, if they exist:

i. $\lim_{x\to 3} \frac{x^2-2x-3}{x-3}$ ii. $\lim_{t\to -2} \frac{t^2-4}{t-2}$

iii. $\lim_{u\to\infty} \frac{5-3u}{6u+1}$ iv. $\lim_{x\to-\infty} \frac{1+2x-x^2}{8x^2+5}$

v. $\lim_{v\to 1} f(u),\ f(u) = \begin{cases} 2u + 3 & for\ u < 1 \\ -u^2 + 2 & for\ u \geq 1. \end{cases}$

b) Determine whether the following functions are continuous at the given point c:

i. $f(x) = \frac{3}{x+4}$ $at\ c = -3$ ii. $g(t) = \begin{cases} \frac{1}{t-2}, & if\ t \neq 2 \\ 3, & if\ t = 2 \end{cases}$ $at\ c = 2$

iii. $h(x) = \begin{cases} x^2 + 1, & if\ x \neq 1 \\ 2, & if\ x = 1 \end{cases}$ $at\ c = 1$ iv. $p(x) = \begin{cases} \frac{x^3-x}{x^2-1}, & if\ x \neq 1 \\ 1, & if\ x = 1 \end{cases}$ $at\ c = 1$

04. a.) Differentiate each of the following functions with respect to x.

i. $f(x) = x^4 - x^2 + 3x - 5$

ii. $g(x) = [x^2 - 2x + 5]^2$

iii. $h(x) = (3x + 4)(x + 2)(2x - 3)$

iv. $q(x) = \frac{x-1}{3x+1}; x \neq -\frac{1}{3}$

v. $R(x) = x^2 \cos x + \tan x - 2x$

b.) Given the information that $f(1) = 2$, $f'(1) = 3$, $g(1) = 4$ and $g'(1) = 5$. Determine the value of the derivative $(fg)'(1)$.

c.) A ball is thrown vertically upwards and after t seconds its height $s(t)$ above th ground is given by $s(t) = 8 + 10t - 5t^2$. Find

i. The height from which the ball is thrown.

ii. The initial velocity of the ball.

iii. The maximum height reached by the ball.